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WORLD MINERAL MARKETS
STAGE II

AN ECONOMETRIC AND SIMULATION APPROACH TO THE WORLD MARKETS FOR COPPER, ALUMINUM, NICKEL AND ZINC

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EXECUTIVE SUMMARY

This monograph presents the second stage of a three stage study of world mineral markets. It differs from the first stage (Mineral Policy Background Paper No. 8) by a) including a review of alternative approaches, b) adding scrap markets to the extent that data are available, c) modifying the treatment of inventories to make it more realistic, d) considering money supply and interest rates as well as capital stock growth and e) including for each mineral a brief description of industry characteristics to facilitate discussion of the implications of some of the results. The third and final stage of this study will provide the computer program itself, together with detailed instructions for its operation, updating and modification. The report on each stage is written so that it can be understood without reference to the other stages. However, readers with very close interests in the commodities traded may wish to compare some of the numerical results presented in this study with those of the previous study to analyse in detail implications of the differences between the first and the second stage of this study. In carrying out the second stage, comments of many readers on the first stage were helpful in many ways and are gratefully acknowledged.

The primary motivation for this study was the fact that most of the existing analyses concerned with the future of metallic mineral commodities (particularly in the case of consumption) were based on rather simple extrapolative models. In such, the future values of production, consumption, and price are projected only on the basis of the historical trend in the variable under consideration. While this approach has the advantage of simplicity, it fails to incorporate the underlying structure of the market for the mineral commodity. Therefore, such models are unable to accomodate changes in the economic environment that may alter the supply and/or demand functions and therefore the trends in production, consumption, and price. Reliance upon such approaches has in the past led to some costly errors.

In contrast, this analysis explicitly incorporates a supply-demand framework for the market under consideration. Furthermore, it recognizes the inherently dynamic nature of the mineral markets as well as the fact that the traditional market clearing equilibrium may not occur. It does, of course, employ historical data in the estimation of the supply and demand equations; thus, it does use time series data. However, the simulation model that will be used to provide predicted future values for production, consumption, and price incorporates the estimated supply and demand functions and, therefore, reflects the structure of the market. With this model, we can consider the effect of changes in the economic environment as well as changes in political or technological conditions. While this model does not provide a perfect crystal ball, it significantly enlarges the analyst's or decision maker's toolbox and provides, through comparison among the many scenarios it can model or through comparison with other approaches, much greater scope for tests of sensitivity which could not be undertaken previously.

In this study we concentrate on four major metallic minerals — copper, aluminum, nickel, and zinc. However, the techniques developed would be applicable to other mineral markets. The essence of our analysis may be summarized as follows:

On the world level, the suppliers and demanders of a mineral commodity may be defined to be countries. However, given the intractability of considering each country separately, the world was divided into seven country groups. Attempting to maintain consistency within the group, these were defined as (1) North America, (2) Central and South America and the Caribbean, (3) Asia, (4) Africa, (5) the European market economies, (6) Oceania, and (7) Centrally Planned Economies. For each of these country groups, we specified and estimated primary supply, secondary (scrap) supply, and demand functions. Since the mineral commodities are traded on a world market, we estimated a single price equation. Once these equations were estimated, they were combined into a simultaneous system to provide the simulation model. In order to obtain predicted future values of production, consumption and price, it was necessary only to input to this simulation model the projected future trends of the exogenous variables (e.g., per capita income). While our "base" simulation assumes that prevailing trends in these exogenous variables will continue in the future, it is also possible to provide simulations based on alternative hypotheses about future trends in these variables and political and technological conditions. Therefore, for each of the minerals considered, we provide several alternative simulations.

From the preceding, it follows that this study is made up of two components — the econometric analysis and the simulation analysis. Following is a brief overview of the techniques employed in each.

THE ECONOMETRIC ANALYSIS

As noted, for each country group, we estimated primary production, secondary production, and demand functions. Then, a price equation was estimated for the entire world market. Briefly, these functions were specified as follows:

Primary Production

In general, the quantity supplied of a commodity would be specified as a function of its price. However, in dealing with minerals one must recognize the dynamic nature of the market. This recognition entails the following: (1) Since known and developed ore bodies are depletable, current extraction depends on the amount previously extracted. (2) Firms extracting a given deposit must maximize profit over time; thus the interest rate becomes a determinant. (3) Since adjustments take a considerable amount of time, it is likely that the producers make production plans on the basis of their *expectations* about future price and future consumption levels. (4) If expectations are not realized, the producers may partially adjust their production plans. These features evolve into a model in which *planned* primary production is a function of *expected* price, *expected* consumption, the amount of the resource previously extracted, and the interest rate. We use a rational expectations approach to the formation of these expectations. Finally, actual primary production is specified to allow for partial adjustment of the production plan if expectations are not realized.

It should be noted that the simulations provided for in this study are meaningful for periods of up to 8 to 10 years. This is less than the lead-time for new major mineral developments that could significantly affect world markets. Entry or exit of major productive capacity or implications of technological break-through in technology are simulated as external shocks to the system.

Secondary Production

Secondary production is made up of new and old scrap production, both of which depend on the price of the mineral. However, new scrap production also depends on current fabricating activity (current consumption) and old scrap production depends on the amounts available for recovery (past consumption). Hence, secondary production is specified as a function of the price of and consumption of the mineral in the current and past periods.

Demand

The demand for a mineral commodity is a derived demand because materials are demanded not for final consumption but rather as inputs in other production processes. Thus, we define quantity demanded (consumption) to be a function of the price of the mineral, the price of substitute or complementary inputs, and (to reflect the demand for final commodities) income in the country group. It is, however, quite likely that the consumers of the minerals — like the suppliers — are unable to adjust instantaneously to changes in market conditions, due to constraints on output scheduling or input mix. Hence, we again employ a specification in which the demanders determine their planned consumption on the basis of expectations and may then adjust this plan partially if expectations are not realized. In this, *planned* consumption is a function of the *expected* price of the mineral, the *expected* price of related commodities, and *expected* income. As with supply, we use a rational expectations approach to the formation of these expectations. Then, we use a partial adjustment framework for actual consumption, to allow for the possibility that the consumers' expectations are not realized.

Price

Given our supply and demand functions, the existence of a world equilibrium would require that all expectations are realized. In such a case, the supply and demand functions could be aggregated and solved simultaneously for the equilibrium (market-clearing) price. In our analysis, we will not require that such an equilibrium be attained. Instead, we allow for the possibility of disequilibrium and assume only that the actual price tends toward this unobservable equilibrium price. In such, we are able to specify a reduced form price equation in which price is a function of all of the exogenous variables in the system.

Each of the econometric equations were estimated individually for the period 1952-1976 using the two-stage least squares technique. No evidence of significant autocorrelation was found; no correction for this problem was necessary.

THE SIMULATION ANALYSIS

As described earlier, our simulation procedure involves the use of the estimated econometric equations to generate projections of the future values of production, consumption, and price. This procedure was itself divided into three phases — development of the simulation model, in-sample predictions to finalize the model, and the future simulations. A brief description of these follows.

The Simulation Model

This model combines an iterative and an extrapolative approach. The iterative feature is the result of our use of the values generated for one period as data to generate projections for the following period. It is extrapolative in the sense that historical trend is used to generate the future values of the exogenous variables that are then used as data in the simulation of the values for the variables under consideration.

We begin with the primary supply, secondary supply, demand, and price equations as estimated for the period 1952-1976. In order to obtain the necessary data for the exogenous variables (e.g., income) for the period 1977-1991, we use simple straight-line extrapolation. We begin with simulations for 1977. We first simulate price, using the available historical data. Then, we simulate consumption in 1977 using our *simulated* 1977 price as data. Then, we simulate primary and secondary production using as data the *simulated* 1977 values of both price and consumption. In order to simulate the 1978 values, the same procedure is employed; however, in such, the 1977 values of production, consumption, and price are required. Hence, we use, as data in the 1978 simulations, the *simulated* values for 1977.

It is this iterative procedure that allows us to make projections several years into the future. We simulate the values of production, consumption, and price in year t and then use these simulated values as data in the simulation of the values in year t+1.

In-Sample Predictions

In the econometric analysis, each of the functions were estimated individually. However, in the simulation model, all of the equations are combined to form a simultaneous equations model. Given this simultaneity, it is then possible that some of the equations, as estimated independently, would prove unsatisfactory in the context of the simulation model.

We therefore performed in-sample predictions to finalize the simulation model. Specifically, using the techniques described above, we simulated production, consumption and price for the period 1965-1977. In this, we employed alternative forms of the individual functions and selected our final simulation model on the basis of "goodness-of-fit", by comparing actual and simulated values.

Future Simulations

Our future simulations are reported for the period 1980-1991. For each of the metallic minerals considered, we first provide a "base" simulation in which it is assumed that the exogenous variables will follow the trend indicated in the period 1962-1976.

Given, however, that the major attribute of this type of approach is its ability to incorporate the effect of changes in the economic, political, or technological environment, we then provide six *illustrative* alternate simulations. In these, we consider both changes in the growth rates of some of the exogenous variables and non-market alterations in production. These examples are intended only to demonstrate the flexibility of our approach; the interested researcher can incorporate any number of alternative assumptions into our basic model to generate alternative simulations. Further, for those simulations presented, we leave the selection of the most appropriate to the reader.

- 1 -

INTRODUCTION

In this analysis, our primary objective was to develop an economic methodology for the examination of the world mineral markets. It is an expansion and refinement of the work carried out during the first stage of this project which was reported in Mineral Background Paper No. 8. Although we will explicitly examine some particular markets — copper, aluminum, nickel, and zinc — we wish to stress the fact that the techniques described would be applicable to other mineral markets. Anyone who wishes to examine, say, silver or lead could use the framework developed here.

Our analysis proceeds in two stages. First, we develop and estimate econometric equations that model the underlying structure of the mineral market. Second, we incorporate these estimated equations into a simulation model that may be used to generate predictions for the future values of primary production, secondary production, consumption, and price for the mineral, under alternative assumptions about world economic conditions.

The purpose of this chapter is two-fold. One purpose is to provide the reader with an overview of the methodology that will be employed. A second purpose is to provide the reader with a "roadmap". We have attempted to structure our analysis in order that it convey information to a wide range of readers. Hence, some of the information is presented at several different levels. Therefore, we will provide several alternative paths through this analysis.

1.1 AN OVERVIEW OF THE METHODOLOGY

Many, or perhaps most, of the preceding studies that have sought to project future values of a particular quantity have simply taken the past values of the quantity, determined the trend line that best fits these values, and projected that line into the future. While this technique has the advantage of simplicity, it does not allow for any analysis of the underlying market structure. Major policy mistakes, both corporate and governmental, resulted from this. Even today many mineral sector issues are still influenced by the results of overoptimistic extrapolations which dominated policy discourse in the past and which led not only to misallocations of investment resources but to unnecessarily heated mineral policy controversies. Neither does a strict time series analysis allow one to examine the results of changes in any of the important variables that effect the market structure. In other words, such an analysis is too simplistic to give much insight into the functioning of the market or the impact of changes in important world economic parameters.

This study employs a basic supply-demand framework in order to analyze the world and its component mineral markets.[1] It does, of course, use historical data to estimate demand and supply equations that will be used in the simulation of future production, consumption, and prices; thus it does use time series data. However, once the equations are estimated, they are explicitly incorporated in the simulation analysis. The basic structure of the market is then incorporated into our simulation model. In this way, our methodology differs from that of a strict time series analysis which would tend to veil the underlying structure.

As described in the preceding, this analysis is divided into two components — the econometric analysis and the simulation analysis. While these two components are not disjoint, it will probably be useful to consider them separately.

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[1] In our analysis, we used as our point of departure the analysis of Fisher, Cootner, and Baily (1972, see Chapter 2 for a survey of this article). As should become clear, however, our analysis differs from theirs in that, in our analysis, we incorporate the dynamic nature of the mineral markets as well as the possibility of a market disequilibrium.

1.1.a THE ECONOMETRIC ANALYSIS

In a traditional supply-demand framework, such as the one we will employ, it is first necessary to define the suppliers and demanders. Since we consider the world market, the suppliers and demanders could be defined to be countries. However, since the number of countries involved make this approach impractical, we define the suppliers and demanders as country groups. Using a criterion of minimizing the number of groups while maintaining internal consistency, the world[2] was subdivided into seven country groups: (1) North America, (2) Central and South America, (3) Asia, (4) Africa, (5) Europe, (6) Oceania, and (7) the Centrally Planned Economies.

For each of the country groups we will specify and estimate a primary supply function, a secondary (scrap) supply function, and a consumption function. Then, since we consider a single world market, we must specify a market equilibrium and the relation between the actual market price and the, perhaps unrealized, equilibrium price. Following is a brief description of the manner in which these functions are specified.

In general, the quantity supplied of a commodity could be specified to be a function of the real price of the commodity. However, the inherently dynamic nature of the production process in mineral extraction necessitates changes in this simple formulation. First, since any given ore body is depletable, current production depends on the amount of the ore body that has been extracted in the past.[3] Second, the dynamic nature of the process requires that the extracting firms maximize profit not simply in a given period but, rather, over the firms' relevant time horizon; thus the interest rate (discount factor) becomes a determinant of supply. Third, adjustments in the extractive industries take a considerable amount of time and it is likely that the producers form production plans on the basis of their expectations about future price and consumption levels. Finally, it is possible that, once market exchanges begin, the producers' expectations are not realized. In this case, we will allow for the possibility that the producers are able to partially adjust to unanticipated price and demand conditions. Summarizing, we will employ a supply function such that *planned* quantity supplied (primary production) is determined by expected price, expected consumption, the amount of the deposit previously extracted, and the interest rate. Then, we will allow the suppliers to partially adjust their primary production [extraction] plans if their expectations are not realized.

In the case of secondary (scrap) production, it is necessary to consider both new scrap (i.e., that created in the fabricating process) and old scrap (i.e., that created through the destruction of existing fabricated products). New scrap production would depend on the real price of the mineral and on current consumption of the mineral (to reflect current fabricating activity). Old scrap production would depend on the price of the mineral and on the availability of fabricated products. Using past consumption of the mineral as a measure of the availability of fabricated products, we then define total secondary production to be a function of the price of the mineral and the consumption of the mineral in the current and past periods.

The demand for a mineral commodity is a derived demand. Minerals are not demanded for final consumption but rather as inputs in other production processes. Thus, we define quantity demanded to be a function of the real price of the mineral, the real price of commodities that are related to the mineral in the production process, and (as a proxy variable reflecting the demand for final products) real per capita income in the country group. It is, however, quite likely that the consumers of the mineral are also faced with the inability to adjust instantaneously to changes in market conditions. Hence, we again employ a specification in which the demanders determine their planned level of consumption on the basis of expectations and then may partially adjust this consumption plan if their expectations are not realized. That is, planned consumption will be specified to be a function of the expected price of the mineral, the expected price of the related commodity, and expected real income. Then, if the expected values differ from the actual values, we provide a specification in which actual consumption can be partially adjusted from the planned consumption in response to the unanticipated market conditions.

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[2] We exclude China, Mongolia, the Democratic Republic of Korea, and the Democratic Republic of Viet Nam for which no data were available.

[3] Depletion of any given orebody does not imply that the world will "run out of resources". This issue is discussed in detail in *The Economics of Mineral Extraction* by G. Anders, W.P. Gramm, S.C. Maurice, and C.W. Smithson, Praeger New York, 1980, particularly Chapter 10.

In the context of the econometric specifications we use for supply and demand, the existence of an equilibrium would require that all expectations are realized. If this occurred, we could aggregate to obtain the world supply and demand functions and then solve these simultaneously to obtain an equilibrium (i.e., market clearing) price. As we will demonstrate, such a solution requires that the price of the mineral be a function of the exogenous variables in the system as well as the lagged values of price, production, and consumption. However, in our analysis, we do not assume that the equilibrium is necessarily achieved. Instead, we assume only that the observed, actual price tends toward the unobservable equilibrium price. Incorporating this, we are then able to specify a reduced form equation for the price of the mineral.

From the preceding sketch, the reader should note that the primary difference between our model and other econometric approaches is that we explicitly incorporate the dynamic nature of the mineral markets. Neither the suppliers nor the demanders may be able to adjust instantaneously to changes in market conditions. Hence, in our analysis, we posit that production and consumption plans (or schedules) are made on the basis of expectations concerning the market conditions that will exist in the future. We will, for the most part, employ a "rational expectations" approach to the manner in which these expectations are formed.[4] Specifically, such an approach assumes that expectations about the future value of a variable are formed not only on the basis of the historical trend that has existed for that variable but also including additional variables, the values of which might be expected to alter the trend. Once exchanges begin to take place, the suppliers and/or demanders may find that their expectations are not realized and may wish to alter the production or consumption schedules. We allow for this possibility by employing a specification that allows partial adjustment of the production and consumption plans if the actual values differ from the expected values. Finally, it should be noted that this model employs disequilibrium analysis. We do not assume that a market-clearing equilibrium is necessarily attained. Instead, we assume only that the market tends toward equilibrium.

1.1.b THE SIMULATION ANALYSIS

In essence, our simulation procedure entails using the estimated econometric equations for primary production, secondary production, consumption, and price to generate projections of the future values of these variables. The techniques we use represent a combination of an iterative and an extrapolative approach. The iterative feature results from our use of the values predicted in one period as data to generate predictions for the next period. Extrapolation is used to predict the future values for the exogenous variables that are then used as data to simulate values for the variables under consideration.

Given the estimated primary supply, secondary supply, demand, and price equations using data from periods t, t-1, t-2 . . ., we wish to project these values for periods t+1, t+2, . . ., t+j. For data, we simply extrapolate the t+j values for the exogenous variables (i.e., the interest rates, real per capita income, etc.) We then proceed to estimate the t+1 values. First we simulate price in period t+1. Then, using the simulated price in period t+1, we simulate consumption. Finally, employing the t+1 simulations for both price and consumption, we simulate primary and secondary supply. The procedure is the same to simulate the t+2 values except that the values simulated for period t+1 are used as data in the simulation of the period t+2 values. For example the t+2 value for price is projected, using, as data, the simulated t+1 values for price, production, and consumption. This again is the iterative feature of our model: We simulate values for period t+i and then use these simulations as data in our simulation of the t+i+1 values.

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[4] In some instances we will employ a simpler "adaptive expectations" approach.

We have attempted to design our simulation model to provide a considerable amount of flexibility. As we noted earlier, we use, as data points for the exogenous variables, the values extrapolated from historical trend. We can, obviously, obtain alternative simulations by making different assumptions about the future paths of these variables. In this way, one may use our basic model to obtain simulations for the future values of production, consumption, and price of a specific mineral under alternative assumptions about world economic conditions (e.g., if the growth rates in real per capita income are lower than those that have existed historically). Furthermore, the model itself can be modified to reflect changes in the structure of the mineral market. For instance, one could simulate the effect on production, consumption, and price of a specific mineral for the case of altered production or consumption by a specific country group due to national policy changes (e.g., if African production decreases by 10% due to political difficulties). To demonstrate this flexibility, we will in our simulations for copper, aluminum, nickel, and zinc first provide a "base" simulation and then provide several alternative scenarios indicating changes in historical trend or changes in structure.

In the case of nickel we illustrate the flexibility of the model by providing a modified base simulation to reflect a possible structural change in the nickel industry from the historic price leadership situation to a competitive market.

1.2 A GUIDE TO THIS RESEARCH

Chapter 2 concludes our overview of the topic area. It contains a description of the various techniques that have been used to examine mineral markets and provides a brief survey of the preceding analyses that we consider most important.

In Part II, we develop our econometric and simulation models. These Chapters, 3 and 4, are of necessity fairly technical and mathematical. We do, however, provide a brief non-technical overview at the beginning of each of these chapters.

Part III of this analysis contains the actual estimations and simulations performed. We begin this section with a brief nontechnical review of our econometric model, Chapter 5. We then, in Chapters 6 through 9, present our estimations for copper, aluminum, nickel, and zinc respectively. Finally, in Chapter 10, we recapitulate our simulation model, describe the simulations that will be performed, and present the results of these simulations for the four minerals considered.

We feel that there are at least three levels at which this research can be approached. For the reader interested in the more technical discussion, we recommend the entire monograph, noting that Chapter 5 and the first part of Chapter 10 are to a great extent simply a review of Chapters 3 and 4. For the reader less interested in the technical derivations, we recommend that he read only the introductions to Chapters 3 and 4 and section 4.1, then proceed to Part III. Finally, it is possible, for that reader interested only in the simulations, to use simply the introduction and section 1 of Chapter 4 and then proceed to Chapter 10.

- 2 -

A REVIEW OF SOME ALTERNATIVE APPROACHES

The purpose of this chapter is to provide an overview of the methods and results which have arisen in the studies of certain metals, including copper, aluminum, nickel, and zinc. In recent years, there has been a proliferation of literature on natural resources in general, stimulated in part by recurring concern (in many quarters) with the spectre of resource exhaustion.

In view of the importance of metals as material inputs into the production process, it is natural that they, too, should have been the focus of at least some empirical investigation by economists and business analysts; and indeed, this has been the case. The future availabilities of these metals (and others as well) is often cited as one of the limiting factors in economic growth. As a result, substantial amounts of resources have been expended in the analysis of the future of these metals. The basic objective of much of this analysis has been to provide forecasts of future market conditions that will determine the availability and price of these materials. It is such studies as these that constitute the subject matter of this survey.

2.1 METHODOLOGICAL PRELIMINARIES

There are, in truth, as many different forecasting techniques as there are forecasts, and it will, therefore, facilitate the analysis to make some preliminary attempt at organizing these alternative forecasting methods. As is the case in any production process, the quality of the output (in this instance, the actual forecast) will likely be no better than the inputs used in the production of the output. Thus, in evaluating a forecast, it is necessary to evaluate the methodological basis from which the forecast was derived. For convenience of exposition, it is useful to arbitrarily dichotomize forecasting methods into two groups — qualitative models and statistical models — although it is clear that the two are not entirely disjoint sets; there is some overlap as will be observed as the discussion proceeds.

Qualitative forecasting methods are difficult to describe or evaluate for the simple reason that there is no uniform model or method which serves as a reference point. That is, there is no model which can be used to replicate the initial forecast with a given set of data, and it is this feature above all others which distinguishes qualitative methods from statistical methods. As one author has aptly stated, a qualitative model is essentially a "rule-of-thumb" technique (see Perlman, 1979), which cannot be used by other investigators to arrive at the same forecast value. One should not infer from this, however, that "rule-of-thumb" forecasts are naive or unsophisticated. Indeed, it is the complexity of this method that makes replication so difficult, since such forecasts are typically based on at least some subjective factors. Even so, the amount of research on which such a forecast is based may be substantial.

Often, qualitative forecasts are, in the final analysis, based on expert opinion. The forecaster will examine the available data, solicit the advice of others — discounting some and heavily weighting some others — then sift through this amalgamation of evidence and formulate a point or interval forecast having given due consideration to the evidence. Exactly what weights to assign to the different and disparate bits and pieces of information is subjectively determined, and, one might add, separates the neophyte from the expert. Experts' qualitative approaches, used for business purposes, have often used such static quantitative models as were available as underpinnings. But experts achieve their higher level of sophistication of analysis and often greater accuracy of forecasts by adding their knowledge of the dynamics of particular markets and knowledge of "the details of time and place" which the builders of formal models typically do not possess. The present study should significantly improve the formal underpinnings for the formulation of expert opinion by providing an expressly dynamic model. When discussing quantitative models it should be stressed that some of the very best and reliable qualitative forecasters in the private sector — who are very highly paid — do not make their forecasts generally available.

A statistical model is any model that uses methods which can be replicated independently of the individual conducting the investigation. That is, the same method can be applied to the same data set by any two individuals (given certain minimal technical skills) with equivalent results. The methods need not be esoteric, only replicable. Expertise and skill is required, of course, in formulating such models and interpreting results. An econometric model is an example of such a statistical model, and enjoys certain advantages over qualitative models.

One of the great advantages of econometric models, in addition to replicability of the results, is that there are reasonably well-defined standards for evaluating the merits of such models and the forecasts generated by these models. The criteria usually include such things as asymptotic standard errors, confidence intervals and other homely but equally intelligible concepts.

An equally or perhaps more important advantage of econometric models is their use in simulation. A simulation model, as the phrase is used here, is simply a difference equation or a set of difference equations. Simulation, then, is the solution to these equations. As an example, consider a simple national income model:

$$
\begin{aligned}
Y_t &= C_t + I_t + G_t \\
C_t &= a_1 + a_2 Y_{t-1} \\
I_t &= b_1 + b_2 (Y_{t-1} - Y_{t-2})
\end{aligned}
$$

Once the parameters a_1, a_2, b_1, and b_2 are known, and a time path for G_t is known, the the simultaneous solution to the above three equations will give the time path of the endogenous variables (C_t, Y_t, and I_t). This is the essence of the simulation process.

Models of this type have found wide application and use in the markets for metals and materials. One reason for the continued interest in such models is that they provide a specific means for assessing the market impact of alternative future values of the endogenous variables, once some future value of the exogenous variable has been specified. As a hypothetical example, a simulation model of the copper market (once the model has been specified and estimated), might provide different forecasts of world copper production under alternative assumptions concerning growth rates in world output or income.

It goes without saying, but is worth repeating, that such models have often missed market turning points due to incorrect specification, structural changes not included in the model, and other reasons. For example, many macroeconometric simulation models which were formulated in the sixties failed to predict major turning points in the economy which occurred during the subsequent decade, in part because the importance of domestic monetary policy and price inflation were not represented in the model. But it was the existence of a specified model that suggested these deficiencies and the modifications needed to correct these deficiencies.

A recent paper from the University of Essex (see Page and Rush, 1976) examined the track record for long-term metal forecasts over the period 1910-1965 and made the following observations (among others):

1. Specificity of forecasts seems to accompany inaccuracy. This is no surprise; it simply means that point forecasts are more likely to miss the mark than interval forecasts.
2. The longer the forecast period is from the present, no matter what forecasting method is used, the greater the error in the forecasts.
3. Practically all forecasters were completely surprised by exogenous shocks (political disruption, strikes, etc.). One might venture the observation that forecasting such shocks is more difficult than forecasting economic variables.
4. Neither qualitative nor quantitative models emerge as the clearly superior technique judged by the accuracy of the forecasts.

Thus, although it is clear that the more technical econometric and statistical models have not proven to be unambiguously superior (on the basis of forecasting ability) to rules-of-thumb models, the formal models are easier to replicate and evaluate. In the strictest sense, a methodological assessment of qualitative models is quite impossible, because the methodological foundations of these models are typically unknown (and perhaps unknowable to an outside observer).

With the above remarks in mind, attention will now be directed to a selective survey of the literature on the markets for copper, aluminum, nickel and zinc. The survey makes no claim to exhausting this voluminous body of literature; such an enterprise is an appropriate subject for a full length book or monograph, not for a survey such as this. In selecting the material to be reviewed here, two criteria have been invoked. First, an attempt has been made to illustrate the different approaches to forecasting supply and demand conditions that have been applied to the materials of primary concern. Thus, both qualitative and statistical models are considered here, although the latter is emphasized since this is the approach we have adopted in our study. Second, we are restricting attention mainly to those studies which have addressed the issue of the future of these particular metals and the market in which they operate.

2.2 SOME QUALITATIVE RESULTS

There has been a vast proliferation of literature on the supply of and demand for metals and minerals during the past ten years. However, of the metals of concern here, it is clear that copper and aluminum have received the lion's share of serious research.

In a recent article, Radcliffe (1976) has noted some key points concerning the world use of material inputs in general, and the use of copper and aluminum more specifically. Observing the concurrent growth of GNP, population, and materials usage since the early 19th century, Radcliffe notes a common trend in the growth rates of these materials and the corresponding growth rates in other demographic and economic variables. This broad view of the historical evidence suggests that the utilization of virtually all metals shows a tendency to grow more rapidly than population and GNP over a period of years, and then to decline to a rate of growth not unlike the growth rate in population. This period of rapid growth is not of uniform length among all metals. For example, steel output grew rapidly for 80 years, reaching a peak growth rate in about 1950.

Radcliffe suggests on the basis of this evidence that the growth rate in copper production has already peaked, and that while the rate of growth of aluminum utilization has not peaked, the most rapid growth of aluminum is past. These conclusions are based on the "intensity-of-use" methodology, about which more will be said at a later juncture.

The main changes identified by Radcliffe as likely influences on future supply and demand of materials are (i) the increasing dependence of many industrialized countries on imports, (ii) a greater uncertainty surrounding future material prices, and (iii) a growing tendency among exporting countries to try to use price and output restriction to gain political concessions in the international political area. Thus, according to the author, "the major materials issues of the near and medium-term future are concerned less with 'running out' of materials physically than with the question of their price and accessibility, and of shifts in the location of the materials industry."

In a very provocative and well-argued paper, Landsberg (1976) has addressed a similar set of issues. In this paper, Landsberg is concerned with an examination of recent trends in the material markets and with certain implications of these trends. He observes that the worldwide increase in per capita consumption has stimulated the development of two important and somewhat contradictory ideas. The first idea is embodied in the *Limits to Growth* arguments: increased consumption could result in materials exhaustion unless remedial action is undertaken. The other is the widespread belief that industrialized economies are in a period of transition, moving away from the production of goods toward the increased production of services.

While acknowledging a kernel of truth in the *Limits* arguments, Landsberg correctly points out, as have Brooks and Andrews (1974), the limited usefulness of distinguishing between renewable and nonrenewable resources in a world where even finite resources are more a flow than a stock. As regards the metals, the key issues are economic and technological: at what price is it feasible to develop new extracting technologies to increase the flow of output of these metals?

The "goods-versus-services" argument, the author observes, is ambiguous at best. Although such an argument is intended to suggest that as economies become more service oriented, materials use will diminish since service industries are assumed to be less materials-intensive. But the service industries, contrary to popular impression, are in fact quite materials-intensive. The services industries include, for example, recreation and medical services, both of which are highly materials-intensive. Moreover, an examination of the data shows that any transitions to services has come only very slowly: during the two decades ending in 1970, services grew from 34 to 38 percent of consumer expenditure — hardly a profound shift. For these reasons, it is misleading to count heavily on such an economic transition to reduce demand for metals by any appreciable amount in the near-term. Unfortunately, it is not clear exactly what conclusions emerge from this analysis, since the author does not go on record with any projection of future production/consumption relationships.

2.3 STRUCTURAL ECONOMETRIC MODELS

In section 2.1 we differentiated between qualitative and statistical models. However, there also exists a dichotomy within the group of statistical models. The structural models are those that attempt to explicitly incorporate the workings of the market in the analysis. On the other hand, time-series models formulate predictions of the future behavior of the variable in question exclusively on the past behavior of that variable.

In this section we will consider some of the structural models that have been proposed. We will return to consideration of the time-series approach in section 2.4.

2.3.a THE INTENSITY-OF-USE APPROACH

One of the more interesting methodological developments in the study of metals centers around the concept of intensity-of-use (I-U) of metals, a concept that has been applied by Brooks and Andrews (1974) and by Malenbaum (1973, 1978). Malenbaum's more recent contribution will be surveyed here, because of the breadth and the substantive content.

Mineral Policy Background Paper No. 4 by Mohide, Warden and Mason combined in its section on nickel forecasting an essentially qualitative analysis, based upon Austrian premises, of post World War II changes in nickel consumption with an informal intensity-of-use analysis. The resulting "most likely" trend forecast of nickel consumption has proved to be quite reliable from 1977 to 1981. In fact estimated 1981 and projected - by industry - 1982 consumption figures will be almost exactly on track. The present 3 stage mineral market study arose in effect from the desire to attempt to formalize the analysis carried out in that study. The following detailed discussion of the intensity-of-use concept will, however, consider Malenbaum's contribution because it is far better known in the literature and it develops the intensity-of-use approach in a formal and comprehensive manner.

Malenbaum's 1978 monograph is a very ambitious study of the world demand for a dozen metals during the next decade — through 1985 — and the next generation — up to the year 2000. The author has two primary objectives, the first of which is to apply the I-U methodology to the demand for metals. The second objective is, of course, to provide estimates of future world demand for 12 metals; aluminum, chrome, copper, iron, manganese, nickel, platinum, steel, tin, tungsten, and zinc. The focus of the study is on demand rather than supply because, according to the author, it is the area of materials demand where there exists a substantial gap in our knowledge. This study builds on the 1973 study by the same author, addressing many of the same issues but yielding some different conclusions.

Malenbaum divides the world into 10 groups, with subtotals for (i) U.S. and non-U.S., and (ii) rich nations and poor nations. The 10 basic consuming groups are: (1) Western Europe (OECD countries in Europe), (2) Japan, (3) other developed countries (Australia, Canada, Israel, N. Zealand, S. Africa), (4) the U.S.S.R., (5) Eastern Europe, (6) Africa, (7) Asia, (8) Latin America, (9) mainland China, and (10) U.S.A. The sum of (6)-(9) constitutes the poor nations.

The basic research strategy is to analyze the intensity-of-use of the material inputs in each of these country groups, where intensity-of-use of material i in year t is given by

$$\text{I-U}_{it} = \frac{D_{it}}{GNP_t} ,$$

and D_{it} is the amount of material i used in year t. Therefore, I-U_{it} is intended to measure the physical amount of i used per unit of GNP in any period.

As Malenbaum acknowledges, there may possibly be some circularity in the statement of the problem as it has been formulated, since there is a question of the direction of casualty between use intensity and physical output. The way the model is formulated assumes that causality runs from GNP to I-U and not vice-versa. It is, of course, possible (and some would say likely) that the relative scarcity of these inputs actually limits and determines the growth rate of final output, but Malenbaum denies this as a likely possibility.* Thus, the fundamental assumptions of the study are that long term growth is not determined by inelastic supplies of any material input, nor is this growth limited by any inelasticities of total output resulting from the finite nature of material inputs.

The research strategy is clear; once projections for GNP_t and D_{it} have been determined, then the values of I-U_{it} follow immediately. What is not entirely clear is the means for determining these future values of the causal variables. Malenbaum says that "the historical record was interpreted on the basis of five-year averages," and again that "projections for 1985 and 2000 were made entirely by estimating two average growth rates (for GNP), one for the ten-plus years from the 1971-75 average to 1985, and the other for the 25-plus years from 1971-75 to 2000." The per-capita GNP estimates were so derived, according to the author. He emphasizes that these 1985 and 2000 data are *not* statistical extrapolations, but exactly what they are is not altogether obvious from the preceeding passages.

Malenbaum's forecasts of world economic growth are summarized in Table 2-1, below:

Table 2-1

World GNP: Annual Growth Rate Forecasts by W. Malenbaum

	1951-75	1975-85	1975-2000
Poor Nations	5.0	3.5	3.4
Rich Nations	4.6	3.5	3.3
Non-U.S. World	5.2	3.5	3.4
World	4.7	3.5	3.3

---

*Actually, one need not necessarily rely on assumptions since there are known tests of causality, e.g., the Sims test (1973). Such a test would be a useful extension of Malenbaum's model, and might also shed some light on the *Limits to Growth* debate.

As is clear from the table, world GNP is expected to grow at an average annual rate of 3.5 percent between 1971-75 and 1985 declining to a rate of 3.3 percent between 1971-75 and 2000, and this in contrast to the actual growth rate of 4.7 percent from 1951-75. So, although total world product is expected to increase by 150 percent between 1971-75 and 2000, these increases will occur at a decreasing rate. This estimate represents a downward revision of Malenbaum's 1973 projections.

In addition, Malenbaum projects an increase in *per capita* GNP in the world overall during the same period, but a decline in per capita GNP in the poor nations as population growth rates tend to outstrip gains in real output in these countries. These projections are much less optimistic than comparable United Nations' projections, as Malenbaum observes. Malenbaum expects the world population growth rate to remain stable through 1985, with moderate declines thereafter.

Malenbaum's basic conclusion to this point is that growth rates in real GNP are expected to decline appreciably during the next two decades, relative to the comparable growth rates for the previous two decades. Nonetheless, per capita GNP is predicted to grow absolutely throughout the world during the rest of this century, largely as a result of "human, quality aspects of the worlds' societies" and not primarily as a result of material inputs. However, the economic gap between rich and poor nations is expected to widen.

The implication for I-U patterns are fairly straightforward. Malenbaum suggests that with the exception of aluminum, world I-U of other materials reached its historical peak a decade or so back, and that the time pattern of I-U for most material inputs follows an inverted U pattern. This projection is quite consistent with Radcliffe's projection which was discussed previously.

For the period 1951-75, the data show declining I-U patterns for virtually all these materials in the U.S., accompanied by steady increases on the part of most poor nations. But in the *very* recent past, the data show a declining I-U pattern even among the developing poor nations. Malenbaum expects this pattern of decline to continue throughout the period of time in question.

Overall, then, Malenbaum anticipates a declining I-U for most materials, and this pattern will be particularly pronounced in the U.S. and other rich nations. The author suggests that this systematic decline is due to reaching some threshold of industrial or economic development among rich nations, and because of the spreading of technological change among poor nations.

Having summarized the background on economic and demographic forecasts which will, according to the author, determine in large measure future I-U of the various metals, it now remains to discuss his projection for the specific materials of interest.

Nickel

Since over 90 percent of total nickel consumption is by steel use, it is not surprising that I-U patterns in steel heavily affect I-U in nickel. Since nickel is used so heavily in the steel purchased by technologically sophisticated industries, it is expected that technologically advanced nations will maintain a high I-U of nickel. And, although I-U is low among poor nations, the author expects to observe increases as new steel-using industries are developed in these countries.

Refined Copper

For the poor nations, I-U more than doubled over the period 1951-55 to 1971-75. For rich nations, I-U increased to a peak and then began to decrease during the same period. Economic development and modernization have contributed to use intensities of this material. Declining use intensities among rich nations can be explained by substitutions of other materials — mainly aluminum — for copper, an effect that is expected to continue into the future, particularly since income elasticities of demand for aluminum are higher than these for copper. Thus, the forecast is of a worldwide decline in the I-U of copper.

Primary Aluminum

Malenbaum expects no absolute declines in the I-U of this metal anywhere, although growth rates in use intensities have slowed steadily in the primary consuming regions. Nonetheless, income growth and industrialization should contribute to increased I-U of primary aluminum.

Zinc

Zinc usage as an anti-corrosive, in die-casting, and in brass and bronze products is expected to continue. But substitution of aluminum for zinc in some uses will contribute to an overall decline in the I-U of zinc.

Quoting the author: "The fundamental force in I-U level probably remains the level of production of goods and services relative to population." Malenbaum discounts the importance of price effects since demand for these inputs is a derived demand. Thus, consumer preferences are not directly reflected in demand for these materials. And, since the price of these materials constitute only a small portion of the finished product price, Malenbaum suggests that price variations will have rather small effects on final product prices, small effects on consumer demand, and, therefore, small effects on the amounts demanded of the material inputs.

His point forecasts of the world demand for these four materials are summarized below:

Table 2-2

World Demand Projections by W. Malenbaum

	1971-75	**2000**	**Ratio**
Nickel (t.m.t.)	618	1314	2.1
Refined Copper (t.m.t.)	7923	16839	2.1
Primary Aluminum (t.m.t.)	12249	36516	3.4
Zinc (t.m.t.)	5506	12022	2.2

Whereas his earlier study projected increased usage of most materials on the order of 3 to 4 times the 1971-75 level by the year 2000, this study gives much more modest forecasts. The projections are closer to a twofold increase for most (with the obvious exception of aluminum). The general expectation is for I-U to be lower in future years than in the recent past for richer nations, but higher for poorer nations. For the world as a whole, I-U is expected to decline during the next 20 years due to the larger proportion of world materials being used by the richer nations.

Malenbaum concludes with the observation that his interpretation of the data does not support the hypothesis that the world is faced with a material exhaustion situation. It is more the quality of human inputs, he contends, and not the quality of material inputs that will determine the limits to economic growth.

The idea of using the intensity-of-use model to forecast future demand conditions is deceptively appealing, particularly in view of the ease of use of the concept. One problem with the concept is that it does not contain a framework suitable for identifying unforseen changes in the underlying economic structure which would stimulate change in output, demand, and therefore, use intensities.

2.3.b A MARKET EQUILIBRIUM APPROACH

Although the econometric literature dealing with commodities, generally, is enormous, there have been relatively few attempts to model the markets for the specific commodities of interest here. To the extent that such models have been developed, virtually all are models of the copper market. As a result, the relevant body of literature is quite small and is primarily concerned with the copper market.

Probably the best known econometric model is represented by a paper on the world copper industry by Fisher, Cootner, and Baily (1972). Since their model is closely related to our own, it will be useful to discuss their work in some detail.

In the most general sense, their model can be described as a multi-equation simulation model of the world supply and demand for copper, with a price adjustment mechanism. Both the supply and demand equations are derived from a Koyck-type, distributed lag model, which follows from the price adjustment mechanism which they specify. Thus, it is necessary to mention this hypothesized price adjustment mechanism.

The authors observe that from the standpoint of copper producers, there are two copper prices. For U.S. producers, there is a U.S. producer price (USPP) and the London Metal Exchange Price (LMEP). They hypothesize that domestic (U.S.) consumers take the USPP as a parameter over which they have no control, and that domestic producers want a stable price policy. Thus, U.S. producers take the existing USPP as given in the short run (SR) and simply decide on the amount to produce at that price. Other countries selling in the U.S. behave similarly.

Their theory of price adjustment is as follows. The difference between U.S. copper consumption and U.S. supplies (primary, secondary and scrap) gives the changes in domestic inventories of copper. Then, if inventories increase *and* the LMEP is less than the USPP, the producers conclude that the USPP is too high and gradually begin to lower it. In contrast, if inventories decrease and the LMEP is greater than the USPP, then producers will tend to raise the price. For the rest of the world, it is assumed that markets operate similarly except that the LMEP is the free market price.

Why are there two prices? Since the USPP has often been well below the LMEP, what factors prevent arbitrage from eliminating this differential? When the LMEP is above the USPP, the data shown that U.S. inventories do get depleted, but very slowly, an observation which confirms the fact that little arbitrage occurs. The reason for so little arbitrage, according to the authors is that copper buyers are unwilling to jeopardize their long-run relationships with producers for the sake of short-run gains from arbitrage.

But this does not explain why producers do not adjust their price upward to realize short-run rewards from the higher price. The authors suggest that this is because of the lag structure in adjustments; notably, that the lag structure is long and adjustments can occur only very slowly. Adjustments are slow on the supply side because of the time required to open new mines and the high investment required to increase output of operating mines. They are slow on the demand side because of the costs associated with substituting aluminum for copper. But because aluminum is a good copper substitute, producers only reluctantly raise the copper price. One implication of these long lags is that the price elasticity of both supply and demand should be higher (in absolute value) in the long run than in the short run.

All of the foregoing suggests a stock-adjustment, distributed lag econometric model, and it is this type of model that is used for estimation. Let

S_t^* = *desired* amount of copper supplied in year t.,
S_t = the actual amount supplied, and
P_t = the prevailing copper price in year t.

Then the desired amount supplied is assumed to be a linear function of P_t, or

$$S_t^* = \alpha_0 + \alpha_1 P_t. \tag{2-1}$$

Assuming that supply adjustments occur with some lag in response to a new price, and that adjustment is incomplete in any year, then it follows that

$$S_t - S_{t-1} = k(S_t^* - S_{t-1}) \quad 0 < k < 1. \tag{2-2}$$

Combining equations (2-1) and (2-2) gives

$$S_t = k\alpha_0 + k\alpha_1 P_t + (1-k)S_{t-1}. \tag{2-3}$$

Repeated lagging of this latter equation and successive substitution gives

$$S_t = \alpha_0 t \, k\alpha_1 \sum_{j=1}^{\infty} \lambda^j P_{t-1}, \quad \lambda = 1-k \tag{2-4}$$

It is, of course, (2-3) that is used for estimation. A first-order autoregressive scheme is assumed, and the model is estimated with a technique due to Fair, an instrumental variables technique which corrects for both simultaneous equation bias and autocorrelation.

Using the above described model and technique a supply function for primary copper (mine production) was fitted for the U.S., Chile, Zambia, Canada, and the Rest-of-World (ROW). The variables used were related according to the following model

$$MP_t^i = \beta_0 + \beta_1 P_t^i + \beta_2 MP_{t-1}^i + U_t \, , \tag{2-5}$$

where

MP_t^i = mine production in year t of the i^{th} country or group of countries.

P_t^i = the corresponding price in year t, and

U_t = the error term.

The estimation results are summarized in Table 2-3 below.

Table 2-3

Primary Copper Supply Functions
Estimated by Fisher, *et. alia.*

Country:	**Coefficient of:**	
	P_t^i	MP_{t-1}^i
U.S.A.	14.27 (2.99)	0.73 (3.55)
Chile	415.4 (2.52)	0.72 (5.51)
Canada	2.13 (2.44)	0.99 (25.11)
Zambia	0.13 (0.28)	1.10 (3.55)
ROW	0.22 (2.32)	0.88 (9.20)

The numbers in parentheses below the coefficient estimates are asymptotic standard errors.

As indicated by these functions, the estimated speed of adjustment is very slow in most cases. For the U.S. and Chile, this adjustment coefficient is roughly one-fourth (1-0.73 for the U.S. and 1-0.72 for Chile) indicating that about 25 percent of the gap between desired and actual production is closed every year. The adjustment speed is slower for Canada and Zambia, on the order of 2 percent for the former country and even lower for the latter. The ROW supply function shows an adjustment speed of 12 percent.

All the primary supply functions exhibit a fairly small short-run price elasticity and a somewhat larger long run elasticity, as indicated below.

Table 2-4

Price Elasticities of Copper Supply from Fisher, *et. alia.*

Country	SR Elasticity*	LR Elasticity*
U.S.	0.453	1.67
Canada	0.188	14.84
Zambia	0.068	not reported
Chile	0.112	0.402
R-O-W	0.196	1.680

*These elasticities are evaluated at the mean.

In the case of secondary (scrap) production, there are only two supply functions estimated, one for the U.S. and one for the Rest-of-World. For the U.S., both an "old scrap" and a "new scrap" supply function were estimated. The price effect was not large for either, completely insignificant for the supply of new scrap, (which depended solely on domestic copper consumption in this model) and very inelastic (about 0.4) for old scrap.

Only a single scrap supply equation was estimated for the R-O-W. For this supply function, the price elasticity is approximately one-fourth, while elasticity with respect to total copper consumption is close to unity.

Demand functions were fitted for the U.S., Europe, Japan, and the ROW. The main determinants of copper demand, in the authors' view, are the own-price of copper, the price of aluminum, and some measure of industrial activity. Once again, the authors use a Koyck-type, distributed lag function to model these demand functions. The basic model, for the ithcountry or group at time t, is given below.

$$D_t^i = \delta_0 + \delta_1 P_{t-1}^i + \delta_2 AP_{t-1}^G + \delta_3 IP_t^i + \delta_4 \triangle ID_t^i + \delta_5 \triangle ID_{t-1}^i + \delta_6 D_{t-1}^i + V_t^i, \qquad (2\text{-}6)$$

where

D_t^i = copper consumption in country i at time t,

P_t^i = own-price of copper at time t,

AP_t^G = German aluminum price at time t,

IP_t^i = index of industrial production at time t,

and

ID_t^i = inventories of durable goods at time t, The model was estimated with ordinary least squares corrected for first-order autocorrelation with the Hildreth-Lu iterative search technique. The results are summarized in the following table:

Table 2-5

World Demand for Copper from Fisher, *et. alia.*

Parameters of:

Consumption	P_{t-1}	AP_{t-1}	IP_t	ΔID_t	ΔID_{t-1}	D_{t-1}	R^2
U.S.	-12.37	8.29	5.08	60.49	-49.40	0.79	0.991
	(-7.06)	(1.79)	(5.56)	(9.43)	(-6.25)	(7.02)	
Europe	-0.27	28.52	9.05	0.54	n.a.	n.a.	0.934
	(-1.37)	(1.14)	(1.56)	(1.60)			
Japan	-0.0002	n.a.	1.72	n.a.	n.a.	n.a.	0.975
	(-1.78)		(24.06)				
ROW	-0.12	0.88	1.97	n.a.	n.a.	0.76	0.939
	(-3.24)	(0.29)	(2.46)			(4.74)	

As the authors observe, the results for the U.S. demand equation are quite good. All coefficients were highly significant with the correct sign. In terms of elasticities, the own-price elasticity of copper consumption is -0.21 in the short run and -0.90 in the long run. The elasticities with respect to the aluminum price are 0.24 in short run and 1.01 in the long run. And with respect to the index of industrial activity, the comparable short-and long-run elasticities are approximately 0.2 and 1.0.

The results for other consuming nations are less satisfactory, partly because of a poorer quality data set. For the European demand equation, the point elasticity estimates are as follows: with respect to copper price, -0.088 in the short run and -0.192 in the long run; with respect to aluminum price, 0.613 in the short run and 1.341 in the long run; and with respect to industrial production, 0.453 in the short run and 0.991 in the long run. These results indicate slower adjustment speed, for a given change in independent variables, than in the U.S.

The Japanese demand results are somewhat different. First, several variables were insignificant. Moreover, no adjustment process is in evidence, indicating that Japan adjusts very quickly to relative price changes or changes in industrial activity. Indeed, the long-and-short-run price elasticities are virtually identical, about -0.10, and the industrial production elasticity is small, 0.60.

The ROW demand equation behaved much as expected with the exception that the aluminum price was found to be asymptotically insignificant. The remaining elasticities are: with respect to copper price, -0.218 in the short run and -0.925 in the long run; with respect to industrial production, 0.409 and 1.74. Note that the price elasticity estimates are similar to the comparable elasticities for the U.S.

In general, the demand results taken as a whole show copper demand to be fairly price inelastic, particularly in the short run. In addition, the level of industrial activity appears to be far and away the most important demand influencing factor.

The most significant fact emerging from this analysis is the "very high (long run) elasticity of supply coupled with low adjustment speeds and relatively low short-run elasticities." Thus, when the long-run equilibrium price is below the current price, there is no pronounced tendency for the price to adjust. Continuous growth in the factors stimulating demand tend to cause prices to systematically remain above the long-run equilibrium price due to the inelasticity of the short-run supply function. Thus, the demand curve appears to be continually shifting rightward, with new mines only slowly coming into production.

Since demand is fairly price inelastic, any sudden decreases in demand can cause significant reductions in the short-run price and contribute, more generally, to a volatile short-run price situation in the copper market.

The authors also illustrate how this model can be used for simulation and forecasting. However, the forecasting performance of the model gives mixed results. Predictably, the model forecasts much better in the near-term than in the longer-term, and the forecasting error grows directly with the length of the forecast period.

2.3.e A DISEQUILIBRIUM ECONOMETRIC MODEL

Dissatisfied with the "equilibrium" assumption which implicity underlies the model by Fisher, et al., MacKinnon and Olewiler, in a very recent unpublished paper, have developed a disequilibrium model of the demand side of the U.S. copper market. As they so indicate, their research is intended to provide an econometric model of the demand for copper under the assumption that the copper market is *not* always in equilibrium, and to correct the statistical bias which results in models which ignore this factor. They do not assume that the copper market is always in disequilibrium, but have identified a large number of periods when they believe that this market was not in equilibrium, and when market exchanges occurred at disequilibrium prices.

During certain periods, they note that U.S. copper producers have held the refined copper price below the market clearing level and have rationed the available supply by means of a non-price mechanism, apparently because producers believe that a stable price policy will maximize profits.*

Since the LMEP is often above the USPP in periods of disequilibrium, it is difficult to specify a market demand function since one cannot readily determine the appropriate price during periods of a significant differential between these two prices. Many studies (e.g., Fisher, et al.) try to circumvent this problem by using a weighted average of the LMEP and USPP.

MacKinnon and Olewiler try to get around this problem by simply using the U.S. price, arguing that foreign copper is an imperfect substitute for domestic copper (which, incidentally, is a testable assertion). If the delivered price is equal, U.S. consumers will prefer domestic copper, and some will prefer domestic copper even at a higher price to build "good will" in the event of future shortages.

The authors identify disequilibrium periods (of excess demand) as those periods when the LMEP was significantly above the USPP. Using quarterly data from 1947-74, they identify 42 observations which they classify as disequilibrium observations. In such periods, the quantity of domestic copper consumed (which is what is reflected in the data) will be less than the quantity demanded, and ordinary least squares estimates will be biased and inconsistent.

Their demand specification, therefore, differs from that in other studies. Their consumption variable excludes consumption of scrap and imported copper not sold at USPP. It includes both the USPP and the LMEP, both deflated by the wholesale price index and lagged by three quarters to capture lagged adjustments on behalf of consumers. Exactly why they settled on a lag of three quarters is not clear in their paper, but clearly, USPP is expected to be negatively related to consumption, and LMEP is expected to be positively related.

In addition, their model includes a measure of industrial activities, the index of industrial production (IP), and a measure for changes in inventories (ΔID). They do not include any measure of aluminum prices, arguing that the aluminum market is often in disequilibrium. And finally, they do not include lagged consumption as an explanatory variable since they suggest that the Koyck lag structure is inappropriate for the copper industry.

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*Note that this observation is consistent with the results from a study on copper pricing by McNicol (1975).

They fitted the model with both OLS and a disequilibrium maximum likelihood estimator. These equations are reproduced below:

OLS:

$$D_t = \underset{(-3.81)}{-2.665}\ \mathrm{USPP}_{t-3} + \underset{(0.69)}{0.256}\ \mathrm{LMEP}_{t-3} + \underset{(10.75)}{5.99}\ \mathrm{IP}_t + \underset{(2.01)}{0.75}\ \Delta \mathrm{ID}_t$$

$$\bar{R}^{2*} = 0.881 \qquad \mathrm{SEE} = 28.32$$

Disequilibrium (Maximum Likelihood):

$$D_t = \underset{(5.28)}{4.57}\ \mathrm{USPP}_{t-3} + \underset{(1.92)}{1.108}\ \mathrm{LMEP}_{t-3} + \underset{(7.44)}{8.77}\ \mathrm{IP}_t + \underset{(7.86)}{1.59}\ \Delta \mathrm{ID}_t$$

$$\bar{R}^2 = \text{n.a.} \qquad \mathrm{SEE} = 26.96$$

If the disequilibrium model is correct, then one would expect coefficient estimates between the two equations to differ, which they clearly do, *and* for the maximum-likelihood estimates to differ between a sample taken strictly from equilibrium observations, and one drawn from disequilibrium periods. This latter hypothesis is borne out on the basis of an F-test.

Unfortunately, the authors do not report elasticity estimates so it is not possible to compare their substantive findings with those reported by Fisher, *et. al.* But, it is clear that in this model, the measure of industrial production appears to be the most important determinant of demand. Finally, it would be very informative to compare the predictive performance of the OLS model with the maximum likelihood model.

2.3.d A SYNTHESIS OF QUALITATIVE AND STRUCTURAL APPROACHES

One of the more comprehensive attempts to summarize and organize the existing state of knowledge of the world copper market is the widely cited book by Banks (1974). In the most general sense, this book can be described as a reasonably complete overview of the world copper market from an economist's perspective. Although the book contains nothing new in the way of econometric evidence, Banks has provided an excellent summary of the state of world copper market.

Basically, the book can be divided into two essentially self-contained sections. The first six chapters provide a comprehensive descriptive analysis of the copper market including some historical background, the structure of the market, the nature of the industry, production and consumption data, and production costs and pricing policy. The second section consists of the last two chapters. These chapters contain discussions of some of the econometric aspects of the demand and supply of copper.

The first two chapters contain some useful background information concerning the division of production and consumption of copper among different countries as well as some comments on the structure of production of copper. There are several facts which Banks points out that are essential to understanding this market. First, although a large amount of mined copper comes from LDC's, the overwhelming preponderance of consumption occurs in the more industrialized countries. Second, the world demand for copper has historically been procyclical: when economic activity is up, demand for copper has historically been on the increase, and vice-versa. Price variations have not been of major significance in explaining changes in copper consumption. Finally, "there is room for only a few firms in this industry" (p. 23).

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*The statistic $\bar{R}^2$ is simply R^2 adjusted for degrees of freedom.

As Banks points out, all of these factors have tended to contribute to collusion among the major producers, and various attempts to form cartels have been observed during this century. As Banks says "through overinvestment and technological progress in the matter of substitutes, producers are always in danger of finding themselves with a chronically depressed market. The temptation to regulate the price is thus always near" (p. 18). A price inelastic demand function coupled with a relatively small number of major producers actually contributes to this tendency.

What Banks observes in the way of market structure is a growing tendency for vertical integration in the copper industry. For many end-users of copper in North America, England, and elsewhere, copper is a very important input, and this in itself tends to promote vertical integration. As it turns out, the Japanese and most European governments have not been unfriendly toward integration. Moreover, the nationalization of the copper industry in some countries has also contributed to their tendency toward vertical integration.

In his discussion of secondary copper production, Banks' central argument is that if secondary copper has been important to the copper market in the past, it may well be crucial in the future. The production of secondary copper has usually been contemporaneously correlated with production of primary copper, but Banks cites some data which suggest that we may be entering a period when the ratio of secondary to total copper production will be increasing. For example, many large purchasers now rate some grades of scrap identical to primary copper for their purposes.

Bank's discussion of the statistical analysis of copper demand primarily centers on some of the problems involved in formulating such a model. The fundamental econometric problem in modeling demand, according to the author, involves the specification of an equation that will capture the dynamic nature of the demand equation, and the impact of price expectation on demand. He suggests, at least implicitly, that the adaptive expectations model is probably the more suitable way of formulating the problem of demand specification, at least in comparison with the partial adjustment approach. The same kind of argument is advanced in his discussion of "some econometric aspects of the supply of copper".

2.4 TIME-SERIES MODELS

In this section, another approach to modeling commodity markets will be introduced and discussed. Although time-series (T-S) methods have been applied extensively in some areas of economics, the methods have hardly been mentioned in connection with the mineral markets.

The feature which distinguishes T-S models from most others is that in the former, predictions of the future behavior of a variable in question is based exclusively on the past behavior of that variable. All T-S models are either stochastic or nonstochastic in nature. Examples of nonstochastic T-S models include the linear trend model, the exponential growth model, the autoregressive trend model, and various types of moving-average models. A fundamental problem with any nonstochastic model is that forecasts from such models have none of the statistical properties that would permit statistical inference. For this reason, such models will not be discussed here. The interested reader can find a very comprehensive discussion of deterministic forecasting models in Montgomery and Johnson (1976).

The assumption underlying the theory of stochastic T-S models is that the time series in question has been generated by a random process, and this process can be described to some extent. Thus, it is assumed that each value in the series—$X_1, X_2, \ldots, X_T$—is randomly selected from a probability distribution —$p(X_1, X_2, \ldots, X_T)$— that assigns probabilities to all possible combinations of $X_1, X_2, \cdots, X_T$. Obviously, the underlying probability distribution function will almost never be completely known, but it may still be possible to model the T-S which explains the randomness in a way that is useful for forecasting.

The simplest, and probably most familiar, kind of stochastic T-S is the *random walk* process. In such a process, it is assumed that each change in X_t is independently selected from a probability distribution with a mean of zero, so that X_t is determined by

$$X_t = X_{t-1} + \xi_t$$

where $X_t - X_{t-1} = \xi_t,$

$$E(\xi_t) = 0$$

$$E(\xi_t, \xi_s) = 0 \text{ for } t \neq s \text{ and } \sigma_\xi^2 \text{ otherwise.}$$

Thus, if the past history of X_T is known and a forecast of $\hat{X}_{T+1}$ is desired, this forecast from a random walk model is simply —

$$\hat{X}_{T+1} = E(\hat{X}_{T+1}/X_T, \ldots, X_1).$$

Since

$$X_{T+1} = X_T + \xi_{T+1},$$

it follows that

$$\hat{X}_{T+1} = X_T + E(\xi_{T+1}) = X_T.$$

Similarly, the forecast two periods ahead is —

$$\begin{aligned} \hat{X}_{T+2} &= E(\hat{X}_{T+1}) + E(\xi_{T+2}) \\ &= E(X_T + \xi_{T+1}) + E(\xi_{T+2}) = X_T, \end{aligned}$$

and, of course, this is the same forecast value of X_{T+k}.

Clearly, this simple random walk process will be useful only in very limited cases. However, it can be shown that the k-period error variance is —

$$\text{var}(\xi_R) = K\sigma_\xi^2,$$

and that statistical inference is possible.

A more general class of T-S models are the linear T-S models developed by Box and Jenkins (1978) among others. In order to understand the underlying conceptual basis of these models, consider a T-S in which successive observations can be represented by a linear combination of independent random variables $\xi_t, \xi_{t-1}, \ldots, \xi_{t-q}$, that are drawn from a normal probability distribution with mean 0 and variance σ_ξ^2, in which case the sequence of the ξ_t random variables is called a "white-noise process", and the underlying T-S model is called a moving average of order q, commonly denoted MA(q). The equation of such a process is —

$$X_i = \mu + \Theta_0 \xi_i - \Theta_1 \xi_{t-1} - \cdots - \Theta_q \xi_{t-q}.$$

The parameters Θ_i can be any sign but it is customary to let $\Theta_0 = 1$.

Thus, by assumption

$$E(\xi_t) = 0$$

$$E(\xi_t^2) = \sigma^2$$

$$E(\xi_t, \xi_{t-k}) = 0 \quad \text{for } t \neq k.$$

Thus, the MA(q) process is described by q + 2 parameters: the mean μ, the variance σ_ξ^2, and the parameters $\Theta_1, \cdots, \Theta_q$.

As it turns out, the series must be stationary (or forced to be stationary by differencing the data) before analysis is possible.

This means, simply, that

$$\sum_{i=1}^{q} \Theta_i \leq k < \infty,$$

— which is trivially met in our example above since the number of Θ_i terms is finite. But one could easily specify the process so that the number of parameters is infinite. There are ways of using the autocorrelation function to test the stationarity condition in any MA(q) process.

The other type of T-S process which is commonly used is the autoregressive process of order p — AR(p). In this process, the current observation, X_t, is assumed to be generated by a weighted average of past observations going back p periods, together with a random disturbance term. The equation of AR(p) is —

$$X_t = \Psi_1 X_{t-1} + \Psi_2 X_{t-2} + \cdots + \Psi_p X_{t-p} + \delta_0 + u_t$$

— where δ_0 is a constant term and u_t is a random error term. If this process is stationary, then the mean of this process, say μ, must be invariant with respect to time, in which case —

$$E(X_t) = E(X_{t-1}) = \cdots = E(X_{t-p}) = \mu.$$

The mean is —

$$E(X_t) = \mu = \Psi_1 \mu + \Psi_2 \mu + \cdots + \Psi_p \mu + \delta$$

— and therefore

$$\mu = \frac{\delta}{1 - \Psi_1 - \Psi_2 - \cdots - \Psi_p}.$$

This latter expression also implies a condition for stationarity, namely, that μ must be finite. Otherwise, the process would drift progressively farther away from any given reference point and could not be stationary. In order for μ to be finite, it is necessary that —

$$\sum_{j=1}^{p} \Psi_j < 1.$$

Many random processes have some of the elements of both MA and AR processes. The extension of the previous analysis that deals with such processes is called the mixed autoregressive — moving average process of order (p, q), or ARMA (p, q), which is represented by the equation —

$$X_t = \Psi_1 X_{t-1} + \cdots + \Psi_p X_{t-p} + \delta + u_t - \Theta_1 \xi_{t-1} - \cdots - \Theta \xi_{t-q} .$$

If it turns out that such a process is nonstationary, then it will be necessary to differentiate the data one or more times to obtain a stationary series. If the data are differentiated exactly d times, then it is said that X_t is an integrated autoregressive — moving average process of order (p, d, q) or simply ARIMA (p, d, q). Any homogeneous nonstationary series can be modeled as an ARIMA (p, d, q) process. The difficulty, of course, is in selecting the values of p, d, and q — that is, in specifying the model. Then, having specified the model, there are the matters of diagnostic checking the model and estimation of the parameters. Fortunately, most economic time series can be modeled as processes with $p \leq 2$ and $q \leq 2$, which greatly simplifies the tasks at hand. For a complete discussion of diagnostic testing and parameter estimation, see Box and Jenkins, 1978.

For the purpose at hand, it is sufficient to note that ARIMA models have proved to be a very powerful forecasting method in many areas of economics and business. As the discussion has progressed to this point, it might be assumed that T-S and econometric models are alternative means of building a model which will enable the analyst to generate forecasts of a particular variable (or set of variables). But it should be clear that it might be possible to combine the two methods, and indeed, this has been shown to be an extremely powerful and accurate forecasting method. That is, one builds a "usual" econometric model to explain the economic relationships among variables, after which T-S methods are used to "explain" the behavior of the residual term from the regression model.

To this point in time, there are evidently no examples of linear T-S models of the markets of concern to us. There is, however, one nonlinear T-S model which has appeared in the literature. In this paper by O'Leary and Butler (1978), an algorithm for forecasting metal prices is presented. They model historical copper price data with Fourier Analysis, since the price data do not follow a linear trend, and the Fourier series allows for periodicities in the data. Since the authors simply present the algorithm, there is no apparent way of evaluating the accuracy of their method.

Nonetheless, the application of T-S methods, particular to metal prices (since price formation models have been so troublesome to model with conventional econometric techniques), would appear to be a very promising means for extending our knowledge of the markets for metallic minerals. It goes without saying that the usefulness of these T-S techniques is entirely an empirical matter.

- 3 -
THE ECONOMETRIC MODEL

In developing the econometric specification, our objective was to provide a model that explicitly incorporates some key elements of the underlying structure of the world mineral markets. It was felt that this could best be accomplished in the context of a traditional supply-demand framework. We will assume that the mineral (as well as related commodities) is traded on a world market; so, we will consider a single world price.[1] We are well aware that a single world price, howsoever derived, only imperfectly reflects the wide range of the prices for different product lines, of differences in payment and delivery terms, and of the practices of discounting. In this world market we define the suppliers and demanders to be country groups. For the purposes of this study, the world (excluding China, North Korea, and North Viet-Nam for which data are not available) is divided into seven country groups:

(1) North America
(2) South America, Central America, and the Caribbean
(3) Asia
(4) Africa
(5) Western Europe
(6) Oceania
(7) Centrally Planned Economies

For each of these country groups it will be necessary to specify a primary supply function, a secondary (scrap) supply function, and a demand function. In order to specify price, we aggregate over the country groups to obtain the market supply function (both primary and secondary) and demand function. Equating these would yield an equilibrium price. Since our specification does not require that equilibrium is necessarily established in every period, we then use a specification for actual price that assumes that the market price tends toward the equilibrium price.

A primary difference between our specification and other structural approaches is that we explicitly incorporate the dynamic nature of the mineral markets. Neither the suppliers nor the demanders are able to adjust instantaneously to changes in market conditions. Hence, we posit that production and consumption plans (or schedules) are made on the basis of expectations concerning the market conditions that will exist in the future. Once exchanges begin to occur in the market period[2] these production and consumption schedules may indeed be altered; but such adjustments would be incomplete.

As we noted in Chapter 1, the formulation of these models is necessarily fairly mathematical. The reader not interested in these techniques may omit the remainder of this chapter and instead read the non-technical review of the models presented in Chapter 5.

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[1] Although our general model is developed in the context of a single world market, we will consider specific features, such as the two-market system that has existed for copper, as they arise.

[2] Data availability limits us to consideration of annual market periods.

3.1 PRIMARY PRODUCTION

We take as our point of departure the most basic specification of a supply function. In such, quantity supplied is defined to be a function of the price of the commodity, the price of the inputs used in the production of the commodity, and the level of available technology. Hence, the quantity supplied of mineral M by a specific country group in period t would be expressed as —

$$Q^{S}_{M,t} = f\,(P_{M,t}, P_{I,t}, T_t). \tag{3-1}$$

This expression can be simplified by using the real (i.e., deflated) price of the mineral to reflect both the price of the mineral commodity and the prices of the inputs as follows:

$$Q^{S}_{M,t} = f\,(P_{M,t}, T_t). \tag{3-2}$$

There exist, however, several features specific to the mineral extracting industries that necessitate modification of this extremely simple specification. Following is a brief description of some of the more important of these features.[3]

Mineral extraction is an inherently dynamic production process. since every orebody is depletable, current production [extraction] depends not only on the level of usage of inputs in the current period but also on the amount of the deposit that has already been extracted. This is due to the fact that an increase in the rate of extraction in any one period increases the cost of extracting a given amount of an ore "reserve" in future periods. Hence, current production must be a function of past production. In our analysis, we have elected to incorporate this feature by including a measure of the remaining deposit. We define this variable to be the total deposit (an unknown constant) minus the total extracted in the preceding periods.

$$TD - \sum_{i=0}^{t-1} Q^{S}_{i}\ .$$

Furthermore, the dynamic nature of the production process requires that the firm maximize profits not simply in the current period but rather over the relevant time horizon. That is, the firm will maximize its present value (or net worth). Thus, the discount rate — the rate of interest — becomes a determinant of current extraction. In this context, an increase in the interest rate (r) causes present profits, that can be reinvested at a higher rate, to increase in value relative to future profits that are more heavily discounted. It follows then that an increase in the interest rate would tend to increase the rate of current extraction ($Q^{S}_{M,t}$).

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[3] For further discussion of the mineral extracting industries, the reader is referred to G. Anders, W.P. Gramm, S.C. Maurice, and C.W. Smithson. *The Economics of Mineral Extraction* (New York: Praeger) 1980.

In the mineral extracting industries, adjustment to changes in demand conditions may involve a considerable amount of time. In the short run, the mineral extracting firm may indeed respond to increased demand by expanding output; but, this expansion is limited to more intensive extraction from existing deposits and constrained by existing shaft or mill capacity. In the longer run, firms can increase output a little more by increasing the number of orebodies worked and by some expansion of physical plant. Finally, given still a longer period of adjustment, the firms may increase exploration and development activities leading to new deposits that may be extracted, or they may implement major technological changes. Such developments will be discussed later as simulations of external shocks to the system. This inability to adjust output instantaneously, coupled with the aforementioned dynamic nature of the production process, implies that the mineral extracting firms make their output decisions on the basis of their expectations of price and demand conditions. More specifically, we will assume in our analysis that the firm selects its profit-maximizing output in period t based on its expectations concerning the price that will exist as well as its expectations of the quantity that will be purchased (i.e., demanded). Thus, we may say that the firm *plans* to extract some quantity, $Q^{S^P}_{M,t}$, on the basis of its expectations concerning price and consumption[4],

$$Q^{S^P}_{M,t} = f\,(P^e_{M,t},\ D^e_{M,t}, \cdots) \tag{3-3}$$

However, there clearly exists the possibility that these expectations will prove invalid. Therefore we will incorporate the fact that, once the firm enters the marketplace, it may revise its expectations — and thereby its production plan — but such adjustment must necessarily be incomplete.

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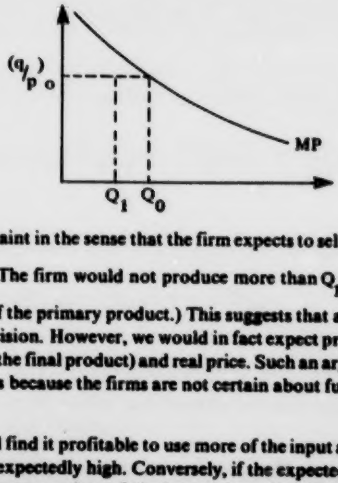
[4] The presence of both expected price and expected demand in equation (3-3) may be unfamiliar, since in a conventional specification only one of them would enter. Our specification is based upon the recent development of disequilibrium market analysis.

We first note that the primary output is itself an input in the production of the refined mineral commodity, In this sense, the primary production decision is predicated on the demand for an input. Let q denote the (unobserved) price of the primary product, distinguished from the price of the refined mineral, p. The competitive firm's input decision (primary production decision) will be determined at that level at which the marginal product of primary production equals the real price, q/p. Given the existing capital stock in refining, the marginal product of primary production will decline.



Suppose that the firm faces a sales constraint in the sense that the firm expects to sell only an amount of the refined mineral requiring Q_1 units of primary production. The firm would not produce more than Q_1 of the primary output. (In this we do not consider the possibility of an inventory of the primary product.) This suggests that a change in the real price, q/p, would have no impact on the primary production decision. However, we would in fact expect primary production (i.e., the input decision) to depend on both sales expectations (of the final product) and real price. Such an argument relies on a speculative demand for the input. This speculative demand arises because the firms are not certain about future sales and therefore future input (primary production) requirements.

If the expected real price is low, firms will find it profitable to use more of the input and to speculate on making extra profits if the demand for the refined mineral is unexpectedly high. Conversely, if the expected real price of primary production is high (i.e., for a given q,p is low), firms will try to use less of the input, since they face the risk of inventory buildups.

Thus, both the demand for final output and the real price of primary output would enter the extracting firm's production decision. Since we can not observe q, we employ the price of final output, p, to reflect this feature.

Finally, it must be noted that the mineral extracting industry holds inventories for the two following reasons: (1) From the traditional inventory-theoretic approach, inventories could be used as a means of compensating for random demand. In such an approach, the size of the inventory would depend on expected sales, the variance in sales, and the costs associated with holding the inventory — including the interest rate. (2) From the dynamic nature of the production process, it is possible that firms will extract and stockpile the resource if they expect higher prices in the future. However, the firm must again consider the interest rate, since this stockpiling represents foregone current profits that could be earning interest.[5] Naturally inventories could accumulate if the firm incorrectly anticipates future price and consumption. It follows from these arguments that inventories will play two roles in our specification. First, the fact that an increase in the rate of interest increases the cost of holding inventory implies that an increase in the interest rate would tend to lower rather than raise the rate of current extraction; thus the overall effect of the interest rate is ambiguous.[6] Second, and more significantly, the unanticipated inventory accumulation resulting from invalid anticipations concerning future price and consumption will play a major role in our specification of the formation of expectations.

Incorporating the preceding discussion, we can respecify our supply function to define *planned* extraction as:

$$Q^{S^P}_{M,t} = f\,[P^e_{M,t}, D^e_{M,t}, T_t, r_t, (TD - \Sigma^{t-1} Q^S_{M,i})] \tag{3-4}$$

where

$Q^{S^P}_{M,t}$ = the *planned* production by a specific country group of mineral M in period t

$P^e_{M,t}$ = the *expected* real price of the mineral in period t

$D^e_{M,t}$ = the expected world consumption [demand] of the mineral in period t

T_t = the level of available technology in period t

r_t = the interest rate in period t

$TD - \Sigma^{t-1} Q^S_{M,i}$ = the remaining amount of the mineral deposit in period t

Employing a linear form, we may rewrite (3-4) as follows:

$$Q^{S^P}_{M,t} = \alpha_0 + \alpha_1 P^e_{M,t} + \alpha_2 D^e_{M,t} + \alpha_3 T_t + \alpha_4 r_t + \alpha_5 (TD - \Sigma^{t-1} Q^S_{M,i}) \tag{3-5}$$

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[5] See Chapter 10 in *The Economics of Mineral Extraction*, op. cit., or *Does Resource Conservation Pay* by G. Anders, W.P. Gramm and C. Maurice, International Institute for Economic Research, Los Angeles, 1978.

[6] As we pointed out earlier, in the absence of inventory considerations, an increase in the interest rate tends to increase current extraction relative to future extraction.

From the preceding discussion, the signs of the parameters would be as follows:

α_0 uncertain

$\alpha_1 > 0$

$\alpha_2 > 0$

$\alpha_3 > 0$

α_4 uncertain

$\alpha_5 > 0$

Since, the total deposit is assumed to be fixed we may rewrite (3-5) as follows:

$$\begin{aligned} Q^{S^P}_{M,t} &= [\alpha_0+\alpha_5 TD] + \alpha_1 P^e_{M,t} + \alpha_2 D^e_{M,t} \\ &\quad + \alpha_3 T_t + \alpha_4 r_t - \alpha_5 \sum^{t-1} Q^S_{M,i} \end{aligned} \tag{3-6}$$

In its present form, equation (3-6) cannot be estimated since it involves (i) planned rather than actual production and (ii) expected rather than actual values for the real price of the mineral and world consumption. We begin with a description of our specification of the expected values.

In the past, the formation of price expectations has traditionally been modeled using a simple autoregressive expectations approach. In such a model, the expected price of the mineral in period t would be formed only on the basis of past prices. The problem with this type of model is that it fails to incorporate additional information available to the supplier. In order to incorporate such information, our analysis will model the formation of price expectations using a rational expectations approach. In this, price expectations are formed on the basis not only of the historical trend in price but also employing additional information that may influence future trends. In the case of the expected real price of the mineral commodity, it would appear that, in addition to past prices, the variable most likely to influence future price would be one that reflects changes in the world inventory level. In our analysis we will use the difference between world primary production and world consumption, $S_{M,t-1} - D_{M,t-1}$, to measure the change in world inventories at the beginning of period t. Using the rational expectations approach, it follows that[7]

$$P^e_{M,t} - P^e_{M,t-1} = \delta_1(P_{t-1} - P^e_{t-1}) + \delta_2(S_{M,t-1} - D_{M,t-1}) \tag{3-7}$$

---

[7] In the earlier version of this analysis (Smithson, *et. alia, op, cit.*), expected price was defined to be —

$$P^e_{M,t} = \beta^S_0 P_{M,t-1} + \beta^S_1 P_{M,t-2} + \cdots + \delta^S X_{t-1}$$

where $\beta^S_1 = \lambda\beta^S_0, \beta^S_2 = \lambda\beta^S_1 = \lambda^2\beta^S_0, \cdots$ and X was a proxy variable to reflect changes in the world stock-consumption ratio. We feel that the difference between primary production and consumption more accurately reflects the changing inventory. As the reader may verify, equation (3-7) is equivalent to —

$$P_{M,t} = \beta^S_0 P_{M,t-1} + \beta^S_1 P_{M,t-2} + \cdots + \delta^S_0 (S_{M,t-1} - D_{M,t-1}) + \delta^S_1 (S_{M,t-1} - D_{M,t-2}) + \cdots$$

where

$$\beta^S_1 = \lambda\beta^S_0, \beta^S_2 = \lambda\beta^S_1 = \lambda\beta^S_2, \cdots \text{ and } \delta^S_1 = \lambda\delta^S_0, \; \delta^S_2 = \lambda\beta^S_1 = \lambda^2\delta^S_0, \cdots$$

In this specification δ_1 represents an adaptive coefficient. If price expectations in period t-1 were too high, they will be revised downward in period t; thus, $\delta_1 > 0$. The coefficient δ_2 reflects the adjustment of expectations to changes in inventory levels. If inventories in the preceding period increase (i.e., $S_{M,t-1} - D_{M,t-1} > 0$) expectations will be revised downward in period t; thus, $\delta_2 < 0$. After some manipulation, we can rewrite equation (3-7) as follows:

$$P^e_{M,t} = \frac{\delta_1}{1-(1-\delta_1)L} P_{M,t-1} + \frac{\delta_2}{1-(1-\delta_1)L} (S_{M,t-1} - D_{M,t-1}) \tag{3-8}$$

In the case of expected world consumption of the mineral commodity we employ a simpler, adaptive expectations formulation. That is,

$$D^e_{M,t} - D^e_{M,t-1} = \delta_1 (D_{M,t-1} - D^e_{M,t-1}) \tag{3-9}$$

In this specification, only the parameter representing the adjustment of expectations is employed. Again, if expected demand in period t-1 exceeded that which was actually forthcoming, we would expect the mineral producer to lower his expectations of consumption in period t; thus, again $\delta_1 > 0$. As with price expectations, we can rewrite (3-9) as follows:

$$D^e_{M,t} = \frac{\delta_1}{1-(1-\delta_1)L} D_{M,t-1} \tag{3-10}$$

We now turn to our specification of the relation between planned and actual production. In this the purpose is to allow the producer some latitude in partially revising his expectations once exchanges begin. To this end, we specify the difference between actual and planned production as follows:

$$Q^S_{M,t} - Q^{S^P}_{M,t} = \gamma_1 (P_{M,t} - P^e_{M,t}) + \gamma_2 (D_{M,t} - D^e_{M,t}) \tag{3-11}$$

The parameters γ_1 and γ_2 represent the adjustment coefficients. If actual price exceeds expected price, the suppliers desire to increase production (i.e., actual production would exceed planned production). The parameter γ_1 measures the degree to which such adjustment is possible; thus, $\gamma_1 \geq 0$. Precisely the same argument holds for γ_2; thus, $\gamma_2 \geq 0$. It must be stressed that adjustment may be impossible (or simply too expensive); therefore, the possibility that γ_1 and/or γ_2 equals zero can not be excluded.[8]

Now, incorporating (3-11) in (3-6), our supply function becomes —

$$\begin{aligned} Q^S_{M,t} = {} & (\alpha_0 + \alpha_5 TD) + \gamma_1 P_{M,t} + (\alpha_1 - \gamma_1) P^e_{M,t} + \gamma_2 D_{M,t} + (\alpha_2 - \gamma_2) D^e_{M,t} \\ & + \alpha_3 T_t + \alpha_4 r_t - \alpha_5 \sum^{t-1} Q^S_{M,i} \end{aligned} \tag{3-12}$$

---

[8] Since we assume that adjustment is incomplete, it follows that γ_1 and γ_2 must be less than one.

Then, using equations (3-8) and (3-10) in (3-12) we may rewrite this equation as

$$
\begin{aligned}
Q^S_{M,t} = {} & (\alpha_0 + \alpha_5 TD) \\
& + \gamma_1 P_{M,t} + (\alpha_1 - \gamma_1)\frac{\delta_1}{(1-(1-\delta_1)L)} P_{M,t-1} + (\alpha_1 - \gamma_1)\frac{\delta_2}{(1-(1-\delta_1)L)} (S_{M,t-1} - D_{M,t-1}) \\
& + \gamma_2 D_{M,t} + (\alpha_2 - \gamma_2)\frac{\delta_1}{(1-(1-\delta_1)L)} D_{M,t-1} \\
& + \gamma_3 T_t + \alpha_4 r_t - \alpha_5 \sum^{t-1} Q^S_{M,i} \\
= {} & (\alpha_0 + \alpha_5 TD)(1-(1-\delta_1)) \\
& + \gamma_1 P_{M,t} + (\delta_1 \alpha_1 - \gamma_1) P_{M,t-1} + (\alpha_1 - \gamma_1)\, \delta_2\, (S_{M,t-1} - D_{M,t-1}) \\
& + \gamma_2 D_{M,t} + (\delta_1 \alpha_2 - \gamma_2) D_{M,t-1} \\
& + \alpha_3 T_t - \alpha_3 (1-\delta_1) T_{t-1} \\
& + \alpha_4 r_t - \alpha_4 (1-\delta_1) r_{t-1} \\
& - \alpha_5 \sum^{t-1} Q^S_{M,i} + \alpha_5 (1-\delta_1) \sum^{t-2} Q^S_{M,i} \\
& + (1-\delta_1)\, Q^S_{M,t-1}
\end{aligned}
\qquad (3\text{-}13)
$$

We can simplify (3-13) somewhat by rearranging terms and using time, t, as a proxy for technical change. Thus,

$$
\begin{aligned}
Q^S_{M,t} = {} & (\alpha_0 + \alpha_5 TD)(1-(1-\delta_1))(\alpha_3(1-\delta_1)) \\
& + \gamma_1 P_{M,t} + (\delta_1 \alpha_1 - \gamma_1) P_{M,t-1} + (\alpha_1 - \gamma_1)\, \delta_2\, ROWS_{M,t-1} \\
& + \gamma_2 D_{M,t} + [\delta_1 \alpha_2 - \gamma_2 - (\alpha_1 - \gamma_1)\, \delta_2]\, D_{M,t-1} \\
& + \delta_3 \alpha_3\, t \\
& + \alpha_4 r_t - \alpha_4 (1-\delta_1)\, r_{t-1} \\
& - \delta_1 \alpha_5 \sum^{t-2} Q^S_{M,i} \\
& + [1-\delta_1 - \alpha_5 + (\alpha_1 - \gamma_1)\delta_2]\, Q^S_{M,t-1}
\end{aligned}
\qquad (3\text{-}14)
$$

— where $ROWS_M$ represents primary production from all country groups other than the one in question.

Then, our estimation equation may be written as follows:

$$
\begin{aligned}
Q^{S}_{M,t} = {} & \theta_0 + \theta_1 P_{M,t} + \theta_2 P_{M,t-1} + \theta_3 \, ROWS_{M,t-1} \\
& + \theta_4 D_{M,t} + \theta_5 D_{M,t-1} + \theta_6 t \\
& + \theta_7 r_t + \theta_8 r_{t-1} \\
& + \theta_9 \sum^{t-2} Q^{S}_{M,i} + \theta_{10} Q^{S}_{M,t-1}
\end{aligned}
\tag{3-15}
$$

In this,

$$
\begin{aligned}
\theta_0 &= (\alpha_0 + \alpha_5 TD)(1-(1-\delta_1))(\alpha_3(1-\delta_1)) && \text{indeterminant} \\
\theta_1 &= \gamma_1 \geq 0 \\
\theta_2 &= \delta_1\alpha_1 - \gamma_1 && \text{indeterminant} \\
\theta_3 &= (\alpha_1 - \gamma_1)\delta_2 && \text{indeterminant} \\
\theta_4 &= \gamma_2 \geq 0 \\
\theta_5 &= \delta_1\alpha_2 - \gamma_2 - (\alpha_1 - \gamma_1)\delta_2 && \text{indeterminant} \\
\theta_6 &= \delta_3\alpha_3 && \text{indeterminant} \\
\theta_7 &= \alpha_4 && \text{indeterminant} \\
\theta_8 &= \alpha_4(1-\delta_1) && \text{indeterminant} \\
\theta_9 &= -\delta_1\alpha_5 < 0 \\
\theta_{10} &= 1-\delta_1 - \alpha_5 + (\alpha_1 - \gamma_1)\delta_2 && \text{indeterminant}
\end{aligned}
$$

In the context of this specification it should again be noted that it may be impossible (or prohibitively expensive) for the producer to adjust his planned production schedule to existing price or consumption conditions. For example, if the producer is unable to adjust production in response to unanticipated price conditions, θ_1 will be zero and $P_{M,t}$ would be excluded from the regression equations. In this instance, it follows that (since $\gamma_1=0$) $\Theta_2=\delta_1\alpha_1>0$ and $\Theta_3=\alpha_1\delta_2<0$. Likewise, if the producer is unable to adjust in response to unanticipated consumption, $\Theta_4=0$ and $D_{M,t}$ would be excluded. If the producer is unable to adjust to unanticipated changes in either variable (i.e., γ_1 and γ_2 equal zero) $\Theta_5=\delta_1\alpha_2-\alpha_1\delta_2>0$.

3.2 SECONDARY PRODUCTION

In this section we will consider the production of the mineral commodity from scrap. As with primary production, the discussion will be in the context of the scrap supply of mineral M by a specific country group. As noted by Fisher, Cootner, and Baily, scrap may be subdivided into two categories. New scrap is that created in the fabricating process itself. Old scrap is that created by the destruction of existing fabricated products. Given the different ways in which the scrap is created, the determinants of these scrap supply functions would be expected to be somewhat different. While our data set precludes the individual estimation of these two supply functions, it is felt that independent examination of these functions will aid the reader in understanding our specification.

New scrap is the simpler of the two. Since, it is created in the fabrication process, it may be specified to be a function of the real price of the mineral and the level of current consumption (a variable that reflects current fabricating activity). That is,

$$
Q^{SCRAP^{NEW}}_{M,t} = f(P_{M,t}, Q^{D}_{M,t}) \tag{3-16}
$$

Employing a linear specification, equation (3-16) may be rewritten as

$$Q_{M,t}^{SCRAP^{NEW}} = \alpha_0 + \alpha_1 P_{M,t} + \alpha_2 Q_{M,t}^{D} \qquad (3\text{-}17)$$

In this the following assumptions are made concerning the signs of the parameters,

α_0 indeterminant

$\alpha_1 > 0$

$\alpha_2 > 0$

Old scrap is more difficult to specify. In very general terms, we know that the quantity of old scrap forthcoming in any period depends on the real price of the mineral commodity and the total amount of the mineral commodity potentially available for recovery,

$$Q_{M,t}^{SCRAP^{OLD}} = f(P_{M,t}, A_{M,t}) \qquad (3\text{-}18)$$

In linear form, equation (3-18) becomes —

$$Q_{M,t}^{SCRAP^{OLD}} = \beta_0 + \beta_1 P_{M,t} + \beta_2 A_{M,t} \qquad (3\text{-}19)$$

where the following assumptions are made concerning the signs of the parameters:

β_0 indeterminant

$\beta_1 > 0$

$\beta_2 > 0$

In this context, the amount of the mineral commodity potentially available for recovery clearly depends on the previous consumption of the mineral commodity by the country group. That is —

$$A_{M,t} = \gamma_1 Q_{M,t-1}^{D} + \gamma_2 Q_{M,t-2}^{D} + \gamma_3 Q_{M,t-3}^{D} + \cdots \qquad (3\text{-}20)$$

Incorporating equation (3-20) in (3-19) and adding this to equation (3-17), total scrap production of commodity M from a specific country group in period t may be specified as —

$$\begin{aligned} Q_{M,t}^{SCRAP} = {} & (\alpha_0 + \beta_0) + (\alpha_1 + \beta_1) P_{M,t} + \alpha_2 Q_{M,t}^{D} \\ & + \beta_2 (\gamma_1 Q_{M,t-1}^{D} + \gamma_2 Q_{M,t-2}^{D} + \gamma_3 Q_{M,t-3}^{D} + \cdots) \end{aligned} \qquad (3\text{-}21)$$

In this equation we have the same type of infinite series that was dealt with in the case of primary production. However, in this case, we do not have the theoretical grounds to *a priori* determine the lag structure. Instead, it will be necessary to use the results of estimations to identify this structure. For example, if we employ the same type of lag structure as that employed in primary production and assume that previously fabricated commodities are available for recovery after two years, it follows that the mineral commodity available for recovery in period t is —

$$A_{M,t} = \gamma_1 Q^{D}_{M,t-1} + \frac{\delta}{1-(1-\delta)L} \gamma_2 Q^{D}_{M,t-2} , \tag{3-22}$$

and equation (3-21) would become

$$\begin{aligned} Q^{SCRAP}_{M,t} &= (\alpha_0 + \beta_0)(1-(1-\delta)) + (\alpha_1 + \beta_1) P_{M,t} - (1-\delta)(\alpha_1 + \beta_1) P_{M,t-1} \\ &+ \alpha_2 Q^{D}_{M,t} + (\beta_2 \gamma_1 - \alpha_2 (1-\delta)) Q^{D}_{M,t-1} \\ &+ \beta_2 (\gamma_2 - (1-\delta) \gamma_1) Q^{D}_{M,t-2} \\ &+ (1-\delta) Q^{SCRAP}_{M,t-1} . \end{aligned} \tag{3-23}$$

For simplicity, equation (3-23) could be rewritten as —

$$\begin{aligned} Q^{SCRAP}_{M,t} = \phi_0 &+ \phi^{P}_1 P_{M,t} + \phi^{P}_2 P_{M,t-1} \\ &+ \phi^{D}_1 Q^{D}_{M,t} + \phi^{D}_2 Q^{D}_{M,t-1} + \phi^{D}_3 Q^{D}_{M,t-2} \\ &+ \phi_S Q^{SCRAP}_{M,t-1} \end{aligned} \tag{3-24}$$

where

$\phi_0 = (\alpha_0 + \beta_0)(1-(1-\delta))$ indeterminant

$\phi^{P}_1 = (\alpha_1 + \beta_1) > 0$

$\phi^{P}_2 = -(1-\delta)(\alpha_1 + \beta_1) < 0$

$\phi^{D}_1 = \alpha_2 > 0$

$\phi^{D}_2 = \beta_2 \gamma_1 - \alpha_2 (1-\delta)$ indeterminant

$\phi^{D}_3 = \beta_2 (\gamma_2 - (1-\delta) \gamma)$ indeterminant

$\phi_S = (1-\delta) > 0$

It could, however, be the case that the lag structure is different from that proposed for primary production. Hence, we generalize equation (3-24) and specify secondary production as —

$$\begin{aligned} Q_{M,t}^{SCRAP} = {} & \phi_0 \\ & + \phi_1^P P_{M,t} + \phi_2^P P_{M,t-1} + \phi_3^P (P_{M,t-2} + \cdots \\ & + \phi_1^D Q_{M,t}^D + \phi_2^D Q_{M,t-1}^D + \phi_3^D Q_{M,t-2}^D + \phi_4^D Q_{M,t-3}^D + \cdots \\ & + \phi_S Q_{M,t-1}^{SCRAP} \end{aligned} \qquad (3\text{-}25)$$

where

ϕ_0 indeterminant

$\phi_1^P > 0$

$\phi_2^P < 0$

$\phi_3^P > 0$

$\vdots$

$\phi_1^D > 0$

ϕ_2^D indeterminant

ϕ_3^D indeterminant

$\vdots$

$\phi_S > 0$

3.3 DEMAND

At the outset, we should stress the fact that the demand for mineral commodities is a *derived* demand. Mineral commodities are demanded not for final consumption but as inputs in other production processes. Thus, the demand for minerals is derived from the demand for final products that use the mineral as an input.

In the simplest form, we might then specify the quantity demanded of mineral commodity M in period t (by a specific country group) as a function of the real price of the mineral commodity, the real price of related commodities, and the demand for those commodities that use mineral M as an input. That is,

$$Q_{M,t}^D = f(P_{M,t}, P_{R,t}, D_{XM,t}). \qquad (3\text{-}26)$$

Even at this level of simplication, a problem is evident. Data is unlikely to be obtainable on the demand for mineral M using final commodities. However, since many of the commodities that use substantial amounts of minerals as inputs are durable goods, the primary determinant of the demand for these final commodities would be income. Hence we will use income in the country group as a proxy variable reflecting the demand for final output. Thus, equation (3-26) becomes —

$$Q_{M,t}^D = f(P_{M,t}, P_{R,t}, Y_t). \qquad (3.27)$$

The difficulty with this simple demand function is that it fails to incorporate the dynamic features of the mineral markets. In section 3.1 we demonstrated that the nature of the mineral markets leads to a specification of the supply function in which the suppliers make their production *plans* on the basis of expectations and then may partially adjust their production schedule if their expectations prove incorrect. In essence, we have argued that the suppliers are unable to adjust instantaneously to changes in market conditions. It is quite possible that the ability of the demanders to adjust is also limited. In the case of the demanders, such an adjustment lag would result if the users of the mineral commodity are unable to instantaneously change either their own output production schedule or their input mix. It follows then that we will use a specification similar to that which we employed for primary supply. That is, we will assume that the demanders determine their planned level of consumption on the basis of expectations. Then, this planned consumption may be partially adjusted if expectations are not realized.

From the preceding, planned consumption in period t would be a function of the expected values of the real price of the mineral commodity, the real price of related commodities, and real income in the country group, i.e. —

$$Q^{D^P}_{M,t} = f(P^e_{M,t}, P^e_{R,t}, Y^e_t). \tag{3-28}$$

Employing a linear form, equation (3-28) may be written as —

$$Q^{D^P}_{M,t} = \alpha_0 + \alpha_1 P^e_{M,t} + \alpha_2 P^e_{R,t} + \alpha_3 Y^e_t \tag{3-29}$$

where the hypothesized signs of the parameters would be —

α_0	uncertain
α_1	< 0
α_2	> 0 if the related input is a substitute < 0 if the related input is a complement
α_3	> 0.

However, if the price of the mineral commodity and the price of the related commodity change by the same proportion (with respect to the final output price), we would expect no substitution effect. This would imply that $\alpha_2 = \alpha_1$. (More specifically, we impose homogeneity of degree zero in $P_{M,t}$ and $P_{R,t}$ on the demand function.)[9] Thus, the planned demand function becomes —

$$Q^{D^P}_{M,t} = \alpha_0 + \alpha_1 (P^e_{M,t} - P^e_{R,t}) + \alpha_3 Y^e_t \tag{3-30}$$

As with the primary supply function, in order to estimate this function, it is necessary to specify (1) the manner in which anticipations are formed and (2) the relation between planned and actual consumption. We turn first to our specifications for the expected values.

---

[9] The point here is that, if, for instance, the prices of both of these commodities doubled, we would not expect the doubling of the price of the related commodity to have an independent effect on the demand for the mineral commodity in question. The inclusion of the price of the related commodity as a determinant of the demand for the mineral commodity in question is to reflect changes in the relative prices of these inputs.

In the case of the price of the mineral commodity, we will use the rational expectations framework described in section 3.1. That is —

$$P^e_{M,t} - P^e_{M,t-1} = \delta_1(P_{t-1} - P^e_{t-1}) + \delta_2(S_{M,t-1} - D_{M,t-1}), \qquad (3\text{-}31)$$

where $\delta_1 > 0$ and $\delta_2 < 0$. It should be noted that, although the formation of expected price follows the same form for both the suppliers and demanders, this does not imply that the price expectations are necessarily equal. In terms of the mathematical equations, δ_1 and δ_2 are not necessarily the same in equations (3-7) and (3-31).

A complication is encountered in the case of the price of related commodities. While it is clear that the demands for mineral commodities are interrelated (e.g., copper and aluminum are substitutes in many applications), incorporation of this interrelation would lead to a simultaneous system of equations which is likely to be intractable. Thus, our specification of the expected price of the related commodity will follow the simpler, adaptive expectations formulation,

$$P^e_{R,t} - P^e_{R,t-1} = \delta_1(P_{R,t-1} - P^e_{R,t-1}). \qquad (3\text{-}32)$$

If the expected price in period t-1 exceeded the actual price, we would expect the demanders to lower their expectations in period t; thus, $\delta_1 > 0$.

In the case of real income, it is clear that historical trend would play a considerable role in the formation of expectations. There exist, however, other variables, changes in which could influence future trend. Two of these are the rate of capital accumulation and the rate of growth of the money supply. If, in the preceding period, the country group accumulated additional physical capital, we would expect real income (or output) to increase in the present period. In the case of the rate of growth of the money supply, the relation is not as clear. However, if the economy is operating below full employment, we would expect an increase in the money supply (or an increase in the rate of growth of the money supply) to lead to an increase in income. Incorporating these factors, we again use a rational expectations framework for the formulation of expected income in the country group. Specifically,

$$Y^e_t - Y^e_{t-1} = \delta_1(Y_{t-1} - Y^e_{t-1}) + \delta_3\dot{K}_{t-1} + \delta_4\dot{M}_{t-1} \qquad (3\text{-}33)$$

where $\dot{K}$ represents the rate of capital acquisition and $\dot{M}$ represents the rate of growth of the money supply. From the preceding discussion, $\delta_1 > 0, \delta_3 > 0,$ and $\delta_4 > 0$.

We next turn to our specification of the relation between planned and actual consumption. As was the case with primary production, we wish to allow the demanders some ability to adjust to new information regarding prices or income. That is, we allow the demanders to partially adjust their planned consumption if their expectations are not realized. To this end, we specify the difference between planned and actual production to be —

$$Q^D_{M,t} - Q^{D^P}_{M,t} = \gamma_1(P_{M,t} - P^e_{M,t}) + \gamma_2(P_{R,t} - P^e_{R,t}) + \gamma_3(Y_t - Y^e_t) \qquad (3\text{-}34)$$

If actual price exceeds expected price, the demanders would desire to decrease consumption and $\gamma_1 < 0$. If the actual price of the related commodity exceeds its expected value, the effect depends on whether the commodity is a substitute or complement for the mineral. If it is a substitute, the demanders would wish more of the mineral commodity and $\gamma_2 > 0$. Conversely, if it is a complement, $\gamma_2 < 0$. If actual income exceeds expected income, the users of the mineral commodity would attempt to expand their own output and consumption of the mineral commodity would increase and $\gamma_3 > 0$.

It should be stressed that these are the expected signs for the adjustment parameters but it is possible for some or all of these to be equal to zero. That is, these parameters represent the degree to which adjustment to unanticipated market conditions is possible and it is quite possible that adjustment is impossible or prohibitively expensive.

As we noted earlier, if the price of the mineral and the price of the related commodity increases by the same proportion, we would expect no substitution effect. This feature again comes into play in equation (3-34). Suppose expected income is equal to actual $(Y_t - Y_t^e = 0)$, then the demand for the mineral commodity is virtually determined by given technology. If $P_{M,t} - P_{M,t}^e$ and $P_{R,t} - P_{R,t}^e$ are proportional, there is no need to alter the input proportions; thus, $\gamma_2 = -\gamma_1$ and equation (3-34) may be rewritten as —

$$Q_{M,t}^D - Q_{M,t}^{D^P} = \gamma_1[(P_{M,t} - P_{R,t}) - (P_{M,t}^e - P_{R,t}^e)] + \gamma_3(Y_t - Y_t^e). \tag{3-35}$$

Finally, we incorporate our specifications of the manner in which expectations are formed and the relation between planned and actual consumption in our demand function (3-30). First, combining (3-35) and (3-30), actual consumption may be specified as —

$$\begin{aligned} Q_{M,t}^D = {} & \alpha_0 + (\alpha_1 - \gamma_1)(P_{M,t}^e - P_{R,t}^e) + \gamma_1(P_{M,t} - P_{R,t}) \\ & + (\alpha_3 - \gamma_3)\,Y_t^e + \gamma_3 Y_t \end{aligned} \tag{3-36}$$

Then we incorporate our specifications of the manner in which expectations are formed. That is, using (3-31), (3-32), and (3-33) in (3-36),

$$\begin{aligned} Q_{M,t}^D = {} & \alpha_0 + \frac{(\alpha_1 - \gamma_1)\delta_1}{1-(1-\delta_1)L}(P_{M,t-1} - P_{R,t-1}) + \frac{(\alpha_1 - \gamma_1)\delta_2}{1-(1-\delta_1)L}(S_{t-1} - D_{t-1}) \\ & + \gamma_1(P_{M,t} - P_{R,t-1}) \\ & + \frac{(\alpha_3 - \gamma_3)\delta_1}{1-(1-\delta_1)L}\,Y_{t-1} + \frac{(\alpha_3 - \gamma_3)\delta_3}{1-(1-\delta_1)L}\,\dot{K}_{t-1} + \frac{(\alpha_3 - \gamma_3)\delta_4}{1-(1-\delta_1)L}\,\dot{M}_{t-1} \\ & + \gamma_3 Y_t \\ = {} & \alpha_0\,(1 - (1-\delta_1)) \\ & + \gamma_1\,(P_{M,t} - P_{R,t}) + (\alpha_1 - \gamma_1)\delta_1\,(P_{M,t-1} - P_{R,t-1}) \\ & + (\alpha_1 - \gamma_1)\,\delta_2\,(S_{t-1} - D_{t-1}) \\ & + \gamma_3 Y_t + (\alpha_3 - \gamma_3)\delta_1 Y_{t-1} + (\alpha_3 - \gamma_3)\delta_3\dot{K}_{t-1} + (\alpha_3 - \gamma_3)\delta_4\dot{M}_{t-1} \\ & + (1 - \delta_1)Q_{M,t-1}^D \end{aligned} \tag{3-37}$$

From this, the estimation equation for quantity demanded of mineral M in period t for a specific country group is —

$$\begin{aligned} Q_{M,t}^D = {} & \Psi_0 + \Psi_1(P_{M,t} - P_{R,t}) + \Psi_2(P_{M,t-1} - P_{R,t-1}) + \Psi_3(S_{t-1} - D_{t-1}) \\ & + \Psi_4 Y_t + \Psi_5 Y_{t-1} + \Psi_6\dot{K}_{t-1} + \Psi_7\dot{M}_{t-1} \\ & + \Psi_8 Q_{M,t-1}^D \end{aligned} \tag{3-38}$$

In this specification,

$$\Psi_0 = \alpha_0 (1 - (1 - \delta_1)) \text{ indeterminant}$$

$$\Psi_1 = \gamma_1 < 0$$

$$\Psi_2 = (\alpha_1 - \gamma_1)\,\delta_1 \quad \text{indeterminant}$$

$$\Psi_3 = (\alpha_1 - \gamma_1)\,\delta_2 \quad \text{indeterminant}$$

$$\Psi_4 = \gamma_3 > 0$$

$$\Psi_5 = (\alpha_3 - \gamma_3)\,\delta_1 \quad \text{indeterminant}$$

$$\Psi_6 = (\alpha_3 - \gamma_3)\,\delta_3 \quad \text{indeterminant}$$

$$\Psi_7 = (\alpha_3 - \gamma_3)\,\delta_4 \quad \text{indeterminant}$$

$$\Psi_8 = (1 - \delta_1) > 0$$

As we have noted previously, it is possible that the demanders are unable to adjust to unanticipated market conditions. That is, γ_1 and/or γ_3 can be equal to zero. If γ_1 is equal to zero, $\Psi_2 = \alpha_1 \delta_1 < 0$ and $\Psi_3 = \alpha_1 \delta_2 > 0$. Likewise if δ_3 is equal to zero, $\Psi_5 = \alpha_3 \delta_1 > 0$, $\Psi_6 = \alpha_3 \delta_3 > 0$, and $\Psi_7 = \alpha_3 \delta_4 > 0$.

3.4 EQUILIBRIUM

In the context of the preceding econometric (stochastic) models, an equilibrium would exist if the suppliers' and demanders' expectations are realized. That is,

$$P^e_{M,t} = P_{M,t}$$

$$P^e_{R,t} = P_{R,t}$$

$$D^e_{M,t} = D_{M,t}.$$

It follows then that, if expectations are realized, planned and actual primary production in country group j would equal —

$$Q^{S^P}_{M,t} = Q^S_{M,t} = \alpha_0 + \alpha_1 P_{M,t} + \alpha_1 D_{M,t} + \alpha_3 t + \alpha_4 r_t + \alpha_5 \sum^{t-1} Q^S_{M,i} \qquad (3\text{-}39)$$

To this, we add secondary (scrap) production from country group j,

$$Q^{SCRAP}_{M,t} = \Phi_0 + \Phi_1 P_{M,t} + \Phi_2 P_{M,t-1} + \Phi_3 Q^D_{M,t} + \Phi_4 Q^D_{M,t-1} + \Phi_5 Q^{SCRAP}_{M,t-1} \qquad (3\text{-}40)$$

Summing the j country groups, we could then specify world production as[10] —

$$WQ^S_{M,t} = \beta_0 + \beta_1 P_{M,t} + \beta_2 P_{M,t-1} + \beta_3 D_{M,t} + \beta_4 (S_{M,t-1} - D_{M,t-1}) + \beta_5 t + \beta_6 r_t + \beta_7 \sum^{t-2} S_{M,i} + \beta_8 WQ^{SCRAP}_{M,t-1} \qquad (3\text{-}41)$$

---

[10] We realize that primary production is ore while secondary production is more similar to refined minerals. Thus there is a problem in summing these two quantities. However, as will become clear in the remainder of this discussion, we intend only to demonstrate the manner in which a reduced form equation for price may be specified. We make no hypotheses about the individual parameters in this equation.

In a similar manner, the realization of expectations implies that we may express the world demand for the mineral as —

$$D_{M,t} = \gamma_0 + \gamma_1 P_{M,t} + \gamma_2 P_{R,t} + \gamma_3 Y_t . \qquad (3\text{-}42)$$

Then, the unobserved equilibrium price may be obtained by equating the supply and demand functions. Using equations (3-41) and (3-42) and requiring that $WQ^{S}_{M,t} = D_{M,t}$, we obtain the following condition:

$$\begin{aligned}
&[\beta_0 + \gamma_0(\beta_3 - 1)] + [\beta_1 + \gamma_1(\beta_3 - 1)]\, P_{M,t} + \beta_2 P_{M,t-1} \\
&+ \beta_4 (S_{M,t-1} - D_{M,t-1}) + \beta_5 t\ \beta_6 r_t \\
&+ \beta_7 \Sigma^{t-2} S_{M,i} + \beta_8 WQ^{SCRAP}_{M,t-1} + \gamma_2 (\beta_3 - 1) P_{R,t} \\
&+ \gamma_3 (\beta_3 - 1)\, Y_t = 0 \qquad (3\text{-}43)
\end{aligned}$$

Solving for the equilibrium price

$$\begin{aligned}
P^{*}_{M,t} = \frac{1}{(1 - \beta_3)\gamma_1 - \beta_1} \quad &[\beta_0 + \gamma_0(\beta_3 - 1)] + \beta_2 P_{M,t-1} + \beta_4 (S_{M,t-1} - D_{M,t-1}) \\
&+ \beta_5 t + \beta_6 r_t + \beta_7 \overset{t-2}{\Sigma}\, S_{M,i} + \beta_8 WQ^{SCRAP}_{M,t-1} \qquad (3\text{-}44) \\
&+ \gamma_2 (\beta_3 - 1) P_{R,t} + \gamma_3 (\beta_3 - 1) Y_t
\end{aligned}$$

We do not in our econometric models assume that this equilibrium position is necessarily obtained. We instead will assume that the actual, observed price tends to adjust toward this equilibrium as

$$P_{M,t} - P_{M,t-1} = \delta\, (P^{*}_{M,t} - P_{M,t-1}). \qquad (3\text{-}45)$$

It follows then that we can express actual price in the following manner:

$$\begin{aligned}
P_{M,t} &= (1 - \delta) P_{M,t-1} + \delta P^{*}_{M,t} \\
&= \eta_0 + \eta_1 P_{M,t-1} + \eta_2 (S_{M,t-1} - D_{M,t-1}) + \eta_3 t \\
&\quad + \eta_4 r_t + \eta_5 \overset{t-2}{\Sigma}\, S_{M,i} + \eta_6 WQ^{SCRAP}_{M,t-1} \qquad (3\text{-}46) \\
&\quad + \eta_7 P_{R,t} + \eta_8 Y_t
\end{aligned}$$

where

$$\eta_0 = \frac{\delta}{(1 - \beta_3)\gamma_1 - \beta_1} [\beta_0 + \gamma_0 (\beta_3 - 1)]$$

$$\eta_1 = (1 - \delta) + \frac{\delta}{(1 - \beta_3)\gamma_1 - \beta_1} \beta_2$$

$$\eta_2 = \frac{\delta}{(1 - \beta_3)\gamma_1 - \beta_1} \beta_4$$

$$\eta_3 = \frac{\delta}{(1 - \beta_3)\gamma_1 - \beta_1} \beta_5$$

$$\eta_4 = \frac{\delta}{(1 - \beta_3)\gamma_1 - \beta_1} \beta_6$$

$$\eta_5 = \frac{\delta}{(1-\beta_3)\gamma_1 - \beta_1} \beta_7$$

$$\eta_6 = \frac{\delta}{(1-\beta_3)\gamma_1 - \beta_1} \beta_8$$

$$\eta_7 = \frac{\delta}{(1-\beta_3)\gamma_1 - \beta_1} \gamma_2 (\beta_3 - 1)$$

$$\eta_8 = \frac{\delta}{(1-\beta_3)\gamma_1 - \beta_1} \gamma_3 (\beta_3 - 1)$$

Given the fact that the coefficients that will be estimated (η_i) are nonlinear combinations of so many of the other parameters, we will make no hypotheses concerning sign. The primary point to be made in this discussion is that price may simply be expressed as a function of the exogenous and predetermined variables in the system.[11]

3.5 ESTIMATION TECHNIQUES

The reader will note that the estimating equations derived in this chapter incorporate endogenous variables as explanatory variables. These include not only lagged values (e.g., $Q^S_{M,t-1}$) but also current values (e.g.,$P_{M,t}$). Clearly, the inclusion of these variables results from the simultaneity of the system. The problem exists, however, that, in this case, traditional ordinary least squares estimation would result in biased estimates of the coefficients; thus, we must employ a simultaneous equations estimation technique. In this study we will employ the two-stage least squares technique treating the lagged endogenous variables as predetermined (i.e., exogenous) variables.

We do, however, encounter a difficulty imposed by the small sample sizes for some of the country groups. Using the two-stage least squares estimation technique to obtain estimates for a market, all of the exogenous (and predetermined) variables relevant for that market should be used as instrumental variables. In some instances the period over which we have data for the country group is simply too short to use all of the variables as instruments (e.g., South America, Asia, and Africa in the case of the demand for aluminum). In such cases, we were required to reduce the number of instrumental variables, concentrating on those variables which, theoretically, are the more significant determinants of the included endogenous variables.

We should also note that we treated the prices of related commodities as exogenous variables. Although it might be more appropriate to treat them as endogenous variables, such a procedure would require the incorporation of the simultaneity of the individual mineral markets (e.g., the copper and aluminum markets would be estimated simultaneously). Such a procedure is simply beyond the scope of this project.

As the reader may be aware, the proposed two-stage least squares estimation technique would be appropriate only if there exists no serial correlation in the error term. In their study of the copper market, Fisher, Cootner, and Baily hypothesized that autocorrelation may well be present. Since the simple Durbin-Watson statistic is biased in the presence of lagged endogenous variables, we employed the method suggested by Hatanaka[12] to test for this autocorrelation. We estimated our equations for several important country groups using this technique. In all cases, serial correlation was not found to be statistically significant. It follows then that the two-stage least squares estimation technique will be acceptable for this project.

Finally, we must stress the fact that the sign predictions made in the preceding sections were made on the basis of the implicit assumption that the explanatory variables are independent. It is quite possible that some of these hypothesized relations may hold due simply to multicolinearity in our data.

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[11] The essence of this discussion is that we are able to specify a reduced form equation for price.

[12] M. Hatanaka, "An Efficient Two-Step Estimator for the Dynamic Adjustment Model with Autoregressive Errors," *Journal of Econometrics*, 1974, 199-220.

- 4 -
THE SIMULATION MODEL

The final objective of this project is to provide simulations of the future values of world price and of production and consumption (both for the world and for individual country groups) for various minerals. In essence, this entails using the econometric models developed in Chapter 3 to provide projections of the future values of these variables. In this chapter we will describe the methodology employed to generate these projections.

We begin with a verbal description of the techniques that will be used. We then turn to a more technical discussion of the manner in which these techniques are implemented. Again, the reader not specifically interested in techniques may wish to omit sections 4.2 and 4.3 and proceed to Chapter 10 in which the simulation model is reviewed and the simulations actually performed in this study are described.

4.1 BASIC TECHNIQUES

The simulation techniques that will be used in this analysis represent a combination of an iterative and an extrapolative approach. The iterative feature results from the fact that the values of the relevant variables in one period are used in our econometric models to generate simulated values for the next period; then, these simulated values are themselves used as data to generate simulations for the next period and so on. The model is extrapolative in the sense that it uses trend to obtain future values of the exogenous variables that may then be used as data in our econometric models to simulate values for the variables under consideration.

Our technique may be summarized briefly as follows. Given the primary supply, secondary supply, and demand functions for each country group and our price formation equation estimated using data for periods t, t-1, t-2, . . ., t-i, we wish to project primary and secondary supply and demand for each country group as well as world production, consumption, and price for periods t+1, t+2, . . ., t+j.

Let us first consider our simulations for period t+1. We begin with the simulation of price, since this equation does not include any current endogenous variables and the price term will be required in the succeeding simulations. To simulate price in period t+1, we use as data the actual values of price, world primary production, world consumption, and world scrap production in period t as well as extrapolated values for the exogenous variables — the interest rate, the price of the related commodity, and income — in period t+1. We then use this simulated price as data in our demand functions to simulate quantity demanded in period t+1. That is, quantity demanded in period t+1 by a specific country group is determined by our simulated price of the mineral commodity in period t+1 in addition to actual values for the price of the mineral, the price of the related commodity, world primary production, world consumption, income, the rates of capital acquisition and growth of the money supply, and quantity demanded in period t and the extrapolated values for the price of the related commodity and income in period t+1. Summing over the country groups we obtain a simulated value for world consumption of the mineral commodity in period t+1. Next, we employ the simulated values of the price of the mineral commodity and quantity demanded in period t+1 to simulate secondary production in period t+1. The scrap production from a specific country group is determined by these variables as well as the actual values in period t of price, consumption, and scrap production.

Again, the world scrap production is obtained simply by summing over the country groups. Finally we incorporate our simulated values for price and world consumption in our estimated primary supply functions to simulate primary production in period t+1. The quantity supplied in period t+1 by a specific country group is determined by the simulated values of the price of the mineral and world consumption in period t+1 as well as the actual values in period t of price, primary supply from the rest of the country groups, consumption, the interest rate and primary quantity supplied and the extrapolated value of the interest rate in period t+1. Then simulated total world primary production is simply the sum of these values for the seven country groups.

In order to simulate values for period t+2, we need to repeat the procedures used to simulate the t+1 values. However, the latter used observable data from period t and, in the case of the simulations for period t+2, we require (unobserved) data from period t+1. Hence, in our simulations for period t+2 we use the simulated values for period t+1 as data. As mentioned earlier, this is the iterative feature of our model. We simulate values for period t+i and then use these t+i simulations as data in the simulation of the t+i+1 values.

The simulation model is designed to provide a considerable amount of flexibility. As we have noted, we use as data points for the exogenous variables — the price of related commodities, the interest rate, income, and the rates of capital accumulation and money stock growth — values extrapolated from historical trends. Clearly, we can alter our simulations by making alternative assumptions concerning the future paths of these variables. Indeed, we can make many alternative simulations to reflect potential changes in the world economic environment and/or changes in the structure of the model itself. For example, an alternative simulation can be obtained assuming that the rate of growth in income (output) in a specific country group is slower or faster than that indicated by historical trend. Or, we can simulate a situation to indicate the results of increased production by a specific country group because of national policy changes rather than market forces. As described in more detail in Chapter 10, we will, for each mineral, provide several alternative simulations. We will first provide a "base" simulation under the assumptions that there is no change in the structure of the model and that the exogenous variables continue to follow historical trend. We will then consider alternative scenarios indicating changes in structure or changes in trend.

4.2 MATHEMATICAL FORMULATION

Given data through period t, we begin by simulating price in period t+1. Using equation (3-46), the price of the mineral commodity is simulated to be

$$\begin{aligned}\tilde{P}_{M,t+1} = {} & \hat{\eta}_0 + \hat{\eta}_1 P_{M,t} + \hat{\eta}_2 (S_{M,t} - D_{M,t}) + \hat{\eta}_3 (t+1) \\ & + \hat{\eta}_4 \tilde{r}_{t+1} + \hat{\eta}_5 \sum^{t-1} S_{M,i} + \hat{\eta}_6 WQ^{SCRAP}_{M,t} \\ & + \hat{\eta}_7 \tilde{P}_{R,t+1} + \hat{\eta}_8 \tilde{Y}_{t+1} \qquad (4\text{-}1)\end{aligned}$$

where the tilda (˜) represents simulated values and the hat (ˆ) above a parameter indicates that this is an estimated value. In this simulation we can use as data the actual values in period t for the price of the mineral ($P_{M,t}$), world primary supply ($S_{M,t}$), world consumption ($D_{M,t}$), and world secondary production ($WQ^{SCRAP}_{M,t}$). However, we also require data for period t+1 on the interest rate (r_{t+1}), the price of the related commodity ($P_{R,t+1}$), and income (Y_{t+1}). Since such data is not available, we project these values for the exogenous variables using historical trends. Specifically, the mean (straight line) rate of growth of the variable will be calculated and used to extrapolate the t+1 values[1], identified with the tilda.

We then use this simulated price in period t+1 to simulate consumption in period t+1. Using equation (3-38), we may simulate quantity demanded by country group j as follows:

$$\begin{aligned}\tilde{Q}^{D}_{M,t+1} = {} & \hat{\Psi}_0 + \hat{\Psi}_1 (\tilde{P}_{M,t+1} - \tilde{P}_{R,t+1}) + \hat{\Psi}_2 (P_{M,t} - P_{R,t}) + \hat{\Psi}_3 (S_{M,t} - D_{M,t}) \\ & + \hat{\Psi}_4 \tilde{Y}_{t+1} + \hat{\Psi}_5 Y_t + \hat{\Psi}_6 \dot{K}_t + \hat{\Psi}_7 \dot{M}_t \\ & + \hat{\Psi}_8 Q^{D}_{M,t}. \qquad (4\text{-}2)\end{aligned}$$

In this we use actual values in period t for the price of the mineral commodity ($P_{M,t}$), the price of the related commodity ($P_{R,t}$), world primary production ($S_{M,t}$), world consumption ($D_{M,t}$), income in the country group (Y_t), the rate of capital accumulation (K_t), the rate of growth of the money supply (M_t), and quantity demanded ($Q^{D}_{M,t}$). As before, we extrapolate the values for the price of the related commodity ($\tilde{P}_{R,t+1}$) and income ($\tilde{Y}_{t+1}$). Given this simulation for each of the country groups we obtain simulated world consumption in period t+1 by simply summing over the j country groups—

$$\tilde{D}_{M,t} = \sum^{j} \tilde{Q}^{D}_{M,t+1} \qquad (4\text{-}3)$$

---

[1] The exogenous variables included in our system of equations are the world price of the related commodity (P_R) and country group specific variables for the interest rate (r), income (Y), the rate of capital accumulation ($\dot{K}$), and the rate of growth of the money supply ($\dot{M}$). For each of these we calculate the mean historical rate of growth (straight line), $\bar{\dot{P}}_R$, $\bar{\dot{r}}$, $\bar{\dot{Y}}$, $\bar{\dot{K}}$, and $\bar{\dot{M}}$. Then the t+1 values of the exogenous variables will be extrapolated as follows:

$$\begin{aligned}\tilde{P}_{R,t+1} &= (1 + \bar{\dot{P}}_R) P_{R,t} \\ \tilde{r}_{t+1} &= (1 + \bar{\dot{r}}) r_t \\ \tilde{Y}_{t+1} &= (1 + \bar{\dot{Y}}) Y_t \\ \tilde{\dot{K}}_{t+1} &= \bar{\dot{K}} \\ \tilde{\dot{M}}_{t+1} &= \bar{\dot{M}}\end{aligned}$$

Then, we use both our simulated price ($\bar{P}_{M,t+1}$) and simulated quantity demanded ($\bar{Q}^{D}_{M,t+1}$) to obtain simulated secondary production in period t+1. Using equation (3-24) scrap production in a specific country group may be simulated as follows:

$$\begin{aligned}\bar{Q}^{SCRAP}_{M,t+1} &= \hat{\Phi}_0 + \hat{\Phi}_1 \bar{P}_{M,t+1} + \hat{\Phi}_2 P_{M,t} \\ &+ \hat{\Phi}_3 \bar{Q}^{D}_{M,t+1} + \hat{\Phi}_4 Q^{D}_{M,t} + \hat{\Phi}_5 Q^{D}_{M,t-1} \\ &+ \hat{\Phi}_6 Q^{SCRAP}_{M,t}\end{aligned} \tag{4-4}$$

The other variables in this equation are of course observable in the data and actual values can be employed. To obtain simulated world scrap production in period t+1 it is necessary only to sum over the j country groups —

$$\bar{WQ}^{SCRAP}_{M,t+1} = \sum^{j} \bar{Q}^{SCRAP}_{M,t+1} \tag{4-5}$$

Next we use our simulated price ($\bar{P}_{M,t+1}$) as well as simulated world consumption ($\bar{D}_{M,t+1}$) to simulate primary supply. Incorporating these into equation (3-15) the primary production in period t+1 may be simulated as —

$$\begin{aligned}\bar{Q}^{S}_{M,t+1} &= \hat{\theta}_0 + \hat{\theta}_1 \bar{P}_{M,t+1} + \hat{\theta}_2 P_{M,t} + \hat{\theta}_3 ROWS_{M,t} \\ &+ \hat{\theta}_4 \bar{D}_{M,t+1} + \hat{\theta}_5 D_{M,t} + \theta_6 (t+1) \\ &+ \hat{\theta}_7 \bar{r}_{t+1} + \hat{\theta}_8 r_t \\ &+ \hat{\theta}_9 \sum^{t-1} Q^{S}_{M,i} + \hat{\theta}_{10} Q^{S}_{M,t}.\end{aligned} \tag{4-6}$$

In this simulation we use the actual values in period t for the price of the mineral ($P_{M,t}$), the primary production from the rest of the world ($ROWS_{M,t}$), world consumption ($D_{M,t}$) the interest rate (r_t), and primary production in period t and earlier periods ($Q^{S}_{M,t-i}$, $i = 0, 1, \cdots$). The value for the interest rate in period t+1 ($\bar{r}_{t+1}$) is again extrapolated from historical trend. Then, total world primary production may be simulated by summing over the j country groups,

$$\bar{S}_{M,t+1} = \sum^{j} \bar{Q}^{S}_{M,t+1} \tag{4-7}$$

Given the simulated values for period t+1, we next turn to simulations for period t+2. In essence, we need to repeat the procedures outlined in equation (4-1) through (4-7) for period t+2. The problem exists that such an approach would require as data the price of the mineral, supply of the mineral, and demand for the mineral in period t+1. Such data are simply not available. Hence, for the simulations in period t+2, we use as data the simulated values for period t+1. For instance, using equation (4-1) to simulate price in period t+2, we need to know the price of the mineral in period t+1. Since this is unknown, we use the previously obtained, simulated value for price in period t+1 ($\bar{P}_{M,t+1}$). Hence, our procedure is iterative, using the t+i simulated values as data to simulate values for period t+i+1.

4.3 SIMULATIONS VS. FORECASTS

The advantage of the iterative procedure described in the preceding section is that it permits the simulation of the variables of interest for numerous periods. There is, however, an inherent problem in this approach. Using an iterative approach rather than simply a single point prediction, the explicit calculation of standard errors for the predicted values becomes virtually intractable.

This problem might best be described using an illustration. Consider, for example, our simulation equation for price, equation (4-1). The simulated value in period t+1($\bar{P}_{M,t+1}$) would have a standard error based on the standard errors of the estimated coefficients ($\bar{\eta}_1$) and, if we treated our extrapolated values of the exogenous variables as random variables, the variation around historical trend for the interest rate and the price of the related commodity. However, the standard error for the simulated value in period t+2($\bar{P}_{M,t+2}$) would be determined not only by these factors but also by the standard errors of the simulated t+1 values used as data. That is, to simulate the price of mineral in period t+2 we will use as data the simulated t+1 values of the price of the mineral($\bar{P}_{M,t+1}$), world primary production ($\bar{S}_{M,t+1}$), world consumption ($D_{M,t+1}$), and world secondary production ($\bar{WQ}^{SCRAP}_{M,t+1}$). Each of these has a standard error that would have to be incorporated in the calculation of the standard error for price in period t+2. Moreover, no data exists on the covariance between the estimated coefficients and the simulated values (e.g., η_2 and $\bar{S}_{M,t+1}$).

In this study we will not attempt to calculate the standard errors for the simulated values; so no confidence intervals will be specified.[2] This lack of confidence intervals is one of the major reasons why we refer to our projections as simulations rather than forecasts.

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[2] It is clear however that the iterative procedure would result in a larger standard error than that which would exist using the simpler point forecast approach.

- 5 -

A NONTECHNICAL REVIEW OF THE ECONOMETRIC MODEL

In this analysis we will use a traditional supply-demand framework in order to incorporate the underlying structure of the world mineral markets. In such, a primary objective was to include the inherently dynamic nature of these markets. We will define both the suppliers and demanders to be country groups. Then, it will be necessary to specify a primary supply function, a secondary (scrap) supply function, and a demand function for each of these country groups. Since we assume that the mineral is traded on a single world market, we consider a single world price and, therefore, we need only to specify a single price formation equation.

In section 5.1 we describe our country groupings and the manner in which the data for the exogenous variables were collected. In sections 5.2 through 5.5 we review the formulation of the primary supply functions, the secondary supply functions, the demand functions, and the price equation that were developed in Chapter 3.

The estimations of the econometric equations described in this chapter will be presented in Chapters 6 through 9. To estimate these equations, we employed the two-stage least squares estimation technique since the equations incorporate endogenous variables as explanatory variables. It should be noted that, although we will make hypotheses concerning the signs of the parameters in the estimation equation, these hypotheses are made assuming that the explanatory variables are independent. As the reader will note in reviewing the estimations, the collinearity that exists in the data can lead to estimates in conflict with our hypotheses.

5.1 THE COUNTRY GROUPINGS

In a traditional supply-demand framework such as the one employed in our analysis, it is necessary to define the suppliers and the demanders. Since we use a world model, the most appropriate specification would be to define the suppliers and demanders as nations. However, given the sheer bulk of data using such an approach, this specification was modified to consider groups of countries.

In specifying the groupings, the criterion used was to limit the number of groups to a manageable size while maintaining consistency within the group. On the basis of this criterion, the world (excluding China, Mongolia, the Democratic Republic of Korea, and the Democratic Republic of Viet Nam for which no data are available) was divided into seven country groups:

(1) North America (Canada and USA)
(2) Caribbean, Central, and South America
(3) Asia (including Israel and Japan)
(4) Africa
(5) European Market Economies
(6) Oceania
(7) Centrally Planned Economies (Albania, Bulgaria, Czechoslovakia, German Democratic Republic, Hungary, Poland, Romania, USSR, and Yugoslavia).

As was described in Chapter 3 and will be shown in the following sections, country group specific data are required for real income, the rate of capital accumulation, the interest rate, and the rate of growth of the money supply. Following is a brief description of these data.

For our income measure we will employ the "index of per capita gross domestic product" from the *United Nations Statistical Yearbook.*

Displayed in Table 5-1, these data were available for the period 1950-1976.

Table 5-1
Index of Per Capita Gross Domestic Product

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1947	.	.	.	.	.	.	.
1948	.	.	.	.	.	.	.
1949	.	.	.	.	.	.	.
1950	0.65	0.60	0.46	0.64	0.47	0.56	0.28
1951	0.70	0.65	0.47	0.65	0.50	0.65	0.30
1952	0.71	0.65	0.49	0.67	0.51	0.63	0.32
1953	0.72	0.65	0.50	0.70	0.53	0.65	0.34
1954	0.70	0.67	0.51	0.72	0.55	0.69	0.37
1955	0.74	0.70	0.53	0.73	0.58	0.70	0.41
1956	0.74	0.71	0.55	0.75	0.60	0.70	0.43
1957	0.74	0.74	0.57	0.76	0.62	0.70	0.47
1958	0.72	0.76	0.58	0.78	0.63	0.72	0.51
1959	0.75	0.76	0.60	0.79	0.65	0.74	0.53
1960	0.77	0.76	0.66	0.79	0.67	0.75	0.58
1961	0.76	0.79	0.67	0.84	0.71	0.74	0.60
1962	0.79	0.80	0.69	0.84	0.74	0.77	0.63
1963	0.83	0.80	0.73	0.83	0.76	0.80	0.65
1964	0.85	0.85	0.76	0.89	0.80	0.84	0.70
1965	0.89	0.86	0.79	0.88	0.82	0.85	0.73
1966	0.93	0.88	0.83	0.88	0.84	0.89	0.73
1967	0.95	0.89	0.86	0.89	0.87	0.89	0.83
1968	0.99	0.92	0.92	0.93	0.91	0.95	0.90
1969	1.00	0.97	0.95	0.97	0.96	0.98	0.93
1970	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1971	1.02	1.04	1.07	1.02	1.02	1.02	1.05
1972	1.07	1.08	1.15	1.04	1.07	1.05	1.09
1973	1.12	1.13	1.22	1.05	1.12	1.10	1.17
1974	1.10	1.18	1.28	1.09	1.13	1.10	1.24
1975	1.07	1.20	1.28	1.08	1.11	1.09	1.29
1976	1.14	1.23	1.33	1.09	1.15	1.11	1.35

To reflect the rate of capital accumulation, we will use the rate of change in the "index of gross fixed capital formation" also obtained from the *United Nations Statistical Yearbook*. These indices are displayed in Table 5-2. As the reader will note, data were not available for the centrally planned economies. Furthermore, this series was considerably shorter than that for income in the case of South America, Asia, and Africa.

Table 5-2
Index of Gross Fixed Capital Formation

Year	North America	South America	Asia	Africa	Europe	Oceania
1947	.	.	.	.	.	.
1948	.	.	.	.	.	.
1949	.	.	.	.	.	.
1950	0.54	.	.	.	0.27	0.35
1951	0.55	.	.	.	0.28	0.37
1952	0.55	.	.	.	0.29	0.39
1953	0.56	.	.	.	0.31	0.41
1954	0.57	.	.	.	0.35	0.44
1955	0.63	.	.	.	0.39	0.46
1956	0.65	.	.	.	0.42	0.45
1957	0.64	.	.	.	0.44	0.48
1958	0.62	.	.	.	0.45	0.50
1959	0.67	.	.	.	0.49	0.54
1960	0.66	0.54	0.46	0.56	0.54	0.57
1961	0.66	0.57	0.47	0.58	0.60	0.58
1962	0.71	0.59	0.48	0.61	0.63	0.61
1963	0.74	0.57	0.53	0.63	0.67	0.68
1964	0.79	0.63	0.59	0.68	0.74	0.77
1965	0.83	0.64	0.65	0.75	0.77	0.81
1966	0.92	0.71	0.71	0.73	0.80	0.83
1967	0.90	0.74	0.76	0.78	0.82	0.85
1968	0.95	0.84	0.86	0.83	0.87	0.91
1969	0.98	0.92	0.94	0.91	0.93	0.95
1970	1.00	1.00	1.00	1.00	1.00	1.00
1971	1.05	1.08	1.11	1.11	1.03	1.01
1972	1.15	1.17	1.29	1.19	1.08	0.98
1973	1.23	1.27	1.42	1.27	1.15	1.03
1974	1.14	1.47	1.65	1.41	1.11	1.05
1975	1.02	1.53	2.12	1.64	1.05	1.08
1976	1.08	1.56	2.50	1.73	1.09	1.12

We were able to obtain interest rate data for only three of the country groups — North America, Europe and Oceania. In general, we obtained our measure of the interest rate in the country groups by calculating the weighted sums of the yields of long-term government bonds for the major countries in the group. These yields were obtained from *International Financial Statistics*. The countries considered were the United States and Canada for North America; Great Britain, France, and West Germany for Europe; and Australia and New Zealand for Oceania. The weights applied to specific countries were calculated as the five-year averages of the ratio of the gross domestic product of the country to the total gross domestic product of the group, where all data were converted to U.S. dollars.[1] The resulting interest rates are presented in Table 5-3.

Table 5-3
Interest Rates

Year	North America	Europe	Oceania
1947	2.26824	4.4992	3.13005
1948	2.46736	4.9662	3.10650
1949	2.33964	5.0988	3.09180
1950	2.34622	5.6490	3.12355
1951	2.60819	6.0771	3.42425
1952	2.74370	6.0280	4.24102
1953	2.97530	5.4924	4.38910
1954	2.56620	5.0089	4.36304
1955	2.82380	5.1695	4.44526
1956	3.09990	5.7333	4.95324
1957	3.57234	6.2964	4.98120
1958	3.52599	5.9938	4.96612
1959	4.19935	5.4226	4.89836
1960	4.14176	5.7963	4.95896
1961	4.00249	5.7889	5.23314
1962	4.07840	5.6219	4.98336
1963	4.12280	5.5292	4.68944
1964	4.26000	5.7759	4.78528
1965	4.33580	6.3494	5.18888
1966	4.83440	6.8767	5.25576
1967	5.08161	6.5051	5.28926
1968	5.55933	6.5809	5.25832
1969	6.42625	7.6960	5.76923
1970	6.94084	8.4761	6.56275
1971	6.18390	8.1640	6.70860
1972	6.12468	7.9464	5.95809
1973	7.16135	9.2747	6.96823
1974	8.13896	11.4083	8.68417
1975	8.26989	10.1431	9.26777
1976	7.99313	9.7298	9.94632

---

[1] The interest rates for West Germany were not available prior to 1956. In order to approximate these rates, we used a regression interpolation. We found the most satisfactory fit to be that using the discount rate (DIS) as the explanatory variable. The following regression equation was estimated for 1956-1977 and used to obtain interest rates for 1947-1955.

$$r_t^{\text{GERMANY}} = 1.913 + 0.561\ \text{DIS}_t + 0.448\ \text{DIS}_{t-1} + 0.284\ \text{DIS}_{t-2}$$

$$R^2 = 0.84$$

While we could have simply used the weighted average for Great Britain and France for the period 1947-1955, we noted that the German interest rates are generally much higher than the other two in the early part of our sample. Consequently, if we used only the weighted average for Great Britain and France, the resulting series would show a large jump between 1955 and 1956 (when German data becomes available).

The rates of growth of the money supply in North America, Europe, and Oceania were calculated in the same manner as the interest rates. In this case, we obtained an index of money growth rates for individual countries from *International Financial Statistics* and then calculated the average rate for the country group.[2] These are presented in Table 5-4.

Table 5-4
Money Growth Rates

Year	North America	Europe	Oceania
1947	2.8290	11.4774	4.6410
1948	-1.3787	14.3540	17.6030
1949	0.1140	14.2375	17.1435
1950	6.2280	9.5570	21.4150
1951	5.3751	11.4415	11.7270
1952	3.7960	8.0854	-4.0920
1953	1.2250	7.9026	13.3292
1954	3.4080	10.0656	3.5302
1955	2.6520	6.8446	1.7950
1956	0.8670	5.4500	0.2212
1957	-0.5788	6.2051	3.8688
1958	2.2643	5.8295	-0.7970
1959	3.0343	8.9145	2.5538
1960	0.2000	7.7809	6.7580
1961	2.2840	8.3719	-3.9942
1962	1.7870	9.4646	1.5664
1963	3.3500	9.6413	3.5384
1964	3.9030	8.1164	7.2544
1965	4.9680	7.2688	0.3384
1966	4.8730	5.4389	1.7856
1967	5.0778	4.4174	5.3051
1968	7.2921	6.1710	6.1487
1969	4.6148	5.9107	7.4903
1970	2.9763	4.9867	5.9698
1971	6.9156	13.0757	5.2926
1972	7.7324	14.1982	12.1460
1973	7.5512	7.8570	24.1048
1974	4.3846	7.4225	4.2152
1975	4.7196	13.1141	12.4284
1976	5.0940	12.7879	13.8065

---

[2] In this case, data for the money supply in West Germany were unavailable prior to 1947 to 1949. Rather than employ a regression interpolation, we simply used the weighted average for Great Britain and France only for the first two years.

5.2 PRIMARY PRODUCTION

In general, the quantity supplied of a particular commodity is specified to be a function of the price of the commodity in question, the price of the inputs used in the production of the commodity, and the level of available technology. We normally simplify this by using the real (deflated) price of the commodity to reflect both the price of the commodity itself and the price of the inputs. However, in the case of the extraction of mineral commodities, this simple formulation must be altered to reflect the dynamic nature of the production process. Following is a brief description of these features we feel are most important.

Since any orebody is depletable, current production [extraction] would depend on the amount of the orebody that has already been extracted. That is, an increase in the rate of extraction in any one period increases the cost of extracting a given amount in future periods; so, increased production in the past tends to reduce current output. In our estimations this feature will be incorporated by including the total extracted in preceding periods($\sum^{t-1} Q^S_{M,i}$) as an explanatory variable.

The fact that we are dealing with a dynamic process implies that the firm will not simply maximize current profits but will instead maximize profit over the relevant time horizon (i.e., maximize its present value or net worth). This requires that the interest rate, r, be a determinant of quantity supplied. If we do not consider inventory holding, an increase in the interest rate tends to increase the rate of current extraction. Conversely, since an increase in the interest rate increases the cost of holding inventories, the effect of an increase in the interest rate might be to decrease the rate of current extraction. Hence, we know that the interest rate will be a determinant of quantity supplied; but, the direction of the effect is not known.

Finally, adjustment to changes in demand conditions may involve a considerable amount of time. Coupled with the dynamic nature of the production process, this feature implies that output decisions are made on the firms' expectations concerning demand conditions. More specifically, we assume that the firms make production *plans* on the basis of their expectations about future price and consumption.

Including the preceding features in our supply function, we define the *planned* primary production of mineral M by a specific country group in period t as —

$$Q^{S^P}_{M,t} = f(P^e_{M,t}, D^e_{M,t}, t, r_t, \sum^{t-1} Q^S_{M,i}). \tag{5-1}$$

In this,

$Q^{S^P}_{M,t}$ = the planned production by a specific country group of mineral M in period t

$P^e_{M,t}$ = the expected real price of mineral M in period t

$D^e_{M,t}$ = the expected world consumption of mineral M in period t

t = time = a proxy for the level of available technology

r_t = the interest rate prevailing in the country group at period t

$\sum^{t-1} Q^S_{M,i}$ = the amount of mineral M previously extracted in the country group

In order to obtain an estimating equation it is necessary to specify (1) the manner in which expectations are formed and (2) the relation between planned and actual production.

In our analysis, we employ a rational expectations form for the formation of price expectations. In such, expectations are formed not only on the basis of historical trend but also including additional information that may alter the trend. In our analysis, we include the difference between world primary production and world consumption (to reflect changes in the world inventory level) as this additional information. In the case of expected consumption we employ a simpler, adaptive expectations formulation.

To specify the relation between planned and actual production, we employ a formulation that allows the producer some latitude in partially revising his expectations once exchanges have begun. We incorporate an adjustment parameter for both price and demand to allow the producers to partially adjust if expectations are not realized.

After incorporating these features, the final supply function in linear form is —

$$\begin{aligned} Q^S_{M,t} = \theta_0 &+ \theta_1 P_{M,t} + \theta_2 P_{M,t-1} + \theta_3 ROWS_{M,t-1} \\ &+ \theta_4 D_{M,t} + \theta_5 D_{M,t-1} + \theta_6 t \\ &+ \theta_7 r_t + \theta_8 r_{t-1} \\ &+ \theta_9 \Sigma^{t-2} Q^S_{M,i} + \theta_{10} Q^S_{M,t-1} \end{aligned} \tag{5-2}$$

where,

$Q^S_{M,t}$ = the actual primary production by a specific country group of mineral M in period t

$P_{M,t}$ = the actual real price of mineral M in period t

$ROWS_{M,t}$ = the primary production from all country groups other than the one in question in period t

$D_{M,t}$ = actual world consumption of mineral M in period t

t = time

r_t = the interest rate prevailing in the country group in period t

We make the following hypotheses concerning the signs of the parameters:

θ_0	indeterminant
θ_1	≥ 0
θ_2	indeterminant
θ_3	indeterminant
θ_4	≥ 0
θ_5	indeterminant
θ_6	indeterminant
θ_7	indeterminant
θ_8	indeterminant
θ_9	< 0
θ_{10}	indeterminant.

It should, however, be noted that the parameters θ_1 and θ_4 reflect respectively the adjustment to unanticipated price and demand conditions. It is possible that adjustment is impossible (or prohibitively expensive). If the producers are unable to adjust production in response to unanticipated price conditions, $\theta_1=0$ and it follows that $\theta_2>0$ and $\theta_3<0$. If the producers are unable to adjust to unanticipated price or consumption conditions ($\theta_1=0$ and $\theta_4=0$) it follows that $\theta_5 > 0$.

5.3 SECONDARY PRODUCTION

Secondary (scrap) production may be subdivided into two categories. New scrap is created in the fabricating process itself, while old scrap is that created by the destruction of existing fabricated products.

New scrap production may be specified to be a function of the real price of the mineral commodity and, to reflect current fabricating activity, the current level of consumption. The amount of old scrap forthcoming depends on the real price of the mineral commodity and the total amount of the mineral commodity available for recovery. In this, the amount available for recovery is, of course, a function of previous consumption levels of the mineral commodity. It follows then that total scrap production of mineral M by a specific country group in period t may be specified as

$$Q^{SCRAP}_{M,t} = f(P_{M,t}, Q^{D}_{M,t}, Q^{D}_{M,t-1}, Q^{D}_{M,t-2}, \ldots) \tag{5-3}$$

where

$Q^{SCRAP}_{M,t}$ = secondary production by a specific country group of mineral M in period t

$P_{M,t}$ = the real price of mineral M in period t

$Q^{D}_{M,t}$ = the consumption in the country group of mineral M in period t.

Using the mathematical rules concerning infinite series and a linear specification, we can transform equation (5-3) into the following estimation equation:

$$\begin{aligned} Q^{SCRAP}_{M,t} = {} & \Phi_0 + \Phi^{P}_1 P_{M,t} + \Phi^{P}_2 P_{M,t-1} + \Phi^{P}_3 P_{M,t-2} \\ & + \Phi^{D}_1 Q^{D}_{M,t} + \Phi^{D}_2 Q^{D}_{M,t-1} + \Phi^{D}_3 Q^{D}_{M,t-2} + \Phi^{D}_4 Q^{D}_{M,t-3} \\ & + \Phi_S Q^{SCRAP}_{M,t-1} \end{aligned} \tag{5-4}$$

In this formulation, the following hypotheses are made about the signs of the parameters:

Φ_0 indeterminant

$\Phi^{P}_1 > 0$

$\Phi^{P}_2 < 0$

$\Phi^{P}_3 > 0$

$\Phi^{D}_1 > 0$

Φ^{D}_2 indeterminant

Φ^{D}_3 indeterminant

Φ^{D}_4 indeterminant

$\Phi_S > 0.$

5.4 DEMAND

The demand for a mineral commodity is a *derived* demand. Mineral commodities are demanded not for final consumption but rather as inputs in other production processes. In this way, the demand for minerals is derived from the demand for those final products that use the mineral as input.

In this context we might then specify the quantity demanded of mineral M by a specific country group in period t as a function of the real price of the mineral commodity, the real price of commodities that are related to the mineral in the final production process, and (as a proxy variable reflecting the demand for final output) real per capita income in the country group. However, we feel that some of the dynamic features discussed in the context of primary production would also be present in the consumption decision. Specifically, we would argue that the demanders are also unable to adjust instantaneously to changes in market conditions; thus, we will employ a specification in which the demanders determine their planned level of consumption on the basis of their expectations and then may partially adjust this plan if their expectations are not realized.

It follows then that we will specify planned consumption as —

$$Q_{M,t}^{D^P} = f(P_{M,t}^{e}, P_{R,t}^{e}, Y_t^{e}) \tag{5-5}$$

where,

$Q_{M,t}^{D^P}$ = planned consumption of mineral M in period t

$P_{M,t}^{e}$ = the expected real price of mineral M in period t

$P_{R,t}^{e}$ = the expected real price of the related commodity in period t

Y_t^{e} = expected real per capita income in the country group in period t

As was the case with primary production, in order to obtain an estimation equation from (5-5) it is necessary to specify (1) the manner in which expectations are formed and (2) the relation between planned and actual consumption.

For the expected real price of the mineral we employ exactly the same rational expectations approach used in primary production. We will posit that price expectations are formed on the basis of both historical trend and changes in the world inventory level (measured by the difference between world primary production and world consumption). We employ the simpler adaptive expectations approach for the formation of the expected price of the related commodity. Finally, for expected real per capita income we use a rational expectations framework in which expectations are formed not only on the basis of the historical trend in income but also incorporating information on the trends in capital accumulation and the growth of the money supply.

As with primary production, we will employ a specification that allows the demanders to partially revise their consumption plans if their expectations are not realized. We incorporate adjustment parameters for the price of the mineral, the price of the related commodity, and income reflecting the degree to which the demanders may adjust their production plans in response to unanticipated market conditions.

Incorporating these specifications (and imposing homogeneity of degree zero in $P_{M,t}$ and $P_{R,t}$), the estimating equation is —

$$\begin{aligned} Q^{D}_{M,t} = {} & \Psi_0 + \Psi_1(P_{M,t} - P_{R,t}) + \Psi_2(P_{M,t-1} - P_{R,t-1}) + \Psi_3(S_{M,t-1} - D_{M,t-1}) \\ & + \Psi_4 Y_t + \Psi_5 Y_{t-1} + \Psi_6 \dot{K}_{t-1} + \Psi_7 \dot{M}_{t-1} \\ & + \Psi_8 Q^{D}_{M,t-1} \end{aligned} \tag{5-6}$$

where

$Q^{D}_{M,t}$ = actual consumption by a specific country group of mineral M in period t

$P_{M,t}$ = actual real price of mineral M in period t

$P_{R,t}$ = actual real price of the related commodity in period t

$S_{M,t}$ = world primary production of mineral M in period t

$D_{M,t}$ = world consumption of mineral M in period t

Y_t = actual real per capita income in the country group in period t

$\dot{K}_t$ = the rate of real capital accumulation in the country group in period t

$\dot{M}_t$ = the rate of growth of the money supply in the country group in period t

We hypothesize the following concerning the signs of the parameters in equation (5-6)

Ψ_0 indeterminant

$\Psi_1 < 0$

Ψ_2 indeterminant

Ψ_3 indeterminant

$\Psi_4 > 0$

Ψ_5 indeterminant

Ψ_6 indeterminant

Ψ_7 indeterminant

$\Psi_8 > 0$

It should be noted that Ψ_1 and Ψ_4 represent the partial adjustment parameters. If the demanders are unable to adjust to unanticipated price conditions, $\Psi_2<0$ and $\Psi_3>0$. Likewise, if the demanders are unable to adjust to unanticipated income conditions, $\Psi_4=0$ and Ψ_5, Ψ_6, and Ψ_7 would be positive.

5.5 MARKET PRICE

In the econometric model we have developed, the existence of an equilibrium would require that the suppliers' and demanders' expectations are realized. In such an instance we could obtain the equilibrium price by (1) summing primary and secondary production for each country group to obtain a total supply function, (2) summing the total supply and demand functions over the country groups to obtain world supply and demand, and (3) solving the world supply and demand functions for the equilibrium (i.e., market clearing) price. In such a solution, the equilibrium price would be a function of the lagged values of price, consumption, and production (both primary and secondary) as well as current values of the exogenous variables — time, and interest rate, the price of related commodity, and income.

In our specification we do not assume that the equilibrium is necessarily achieved. Instead we assume only that the actual (observed) price tends toward this unobservable equilibrium price. Incorporating this, our estimation equation is —

$$\begin{aligned} P_{M,t} = {} & \eta_0 + \eta_1 P_{M,t-1} + \eta_2 P_{M,t-2} + \eta_3 (S_{M,t-1} - D_{M,t-1}) + \eta_4 t \\ & + \eta_5 r_t + \eta_6 \sum^{t-2} S_{M,i} + \eta_7 WQ^{SCRAP}_{M,t-1} \\ & + \eta_8 P_{R,t} + \eta_9 P_{R,t-1} + \eta_{10} P_{R,t-2} + \eta_{11} Y_t \end{aligned} \tag{5-7}$$

where

$P_{M,t}$ = the real price of mineral M in period t

$S_{M,t}$ = world primary production of mineral M in period t

$D_{M,t}$ = world consumption of mineral M in period t

r_t = the interest rate in period t

$WQ^{SCRAP}_{M,t}$ = world secondary production of mineral M in period t

$P_{R,t}$ = the real price of the related commodity in period t

Y_t = income in period t.

In essence, equation (5-7) represents the reduced form equation for price. In this instance we make no hypotheses concerning the signs of the parameters. Finally it should be noted that, in this specification, r_t, and Y_t should represent the world interest rate and world per capita income. Given the aggregation problems involved, we will use in our estimations the North America values for these variables.

- 6 -
COPPER

6.1 AN OVERVIEW OF THE COPPER MARKET

This and subsequent "Overview" sections are written for the non-technical reader. They view time periods longer than those used in the econometric analysis and consider a number of exogenous factors. They may therefore at times be at variance with the subsequent technical discussion, for which they are intended to provide a broader perspective. The figures in the Appendix are intended to provide some of the perspective within which the computer simulations have to be interpreted.

Copper market trends are used by many as a barometer of industrial activity in the developed world, reflecting the ups and downs of the world economy. On the other hand copper prices have long been notorious for their short-run volatility.

A look at the pattern of copper production and consumption throughout this century shows from 1900 to 1914 a slow but gradual increase. There was a sharp rise occasioned by World War I, followed in turn by the sharp post World War I recession. Next came the boom of the 1920's followed by the recession from 1929 to 1932. This is succeeded by a prolonged and rapid increase in consumption as the world geared up for World War II, followed by the sharp post World War II recession. In the mid 40's began the longest sustained and most spectacular boom period.

While the 1929 peak in production and consumption exceeded the World War I peak by about one third and the World War II peak exceeded the World War I peak by about 100%, the 1974 peak was over two and a half times higher than the World War II peak. After a steep drop in the 1975-76 recession, which in absolute terms paralleled the drop of the 1929-1932 recession, the steep upward movement in consumption resumed for another two years.

If one compares the movements in physical production and consumption with the movements of real copper prices, however, a somewhat different picture emerges. The period from 1900 to the beginning of World War I showed on the whole a declining price trend. While the World War I boom and the subsequent recession in production and consumption were reflected in real price movements, during the 1920's boom, even though production and consumption picked up, prices continued to decline, to start rising only during the very end of that boom period. During the 1929-1932 recession production, consumption and prices moved down in parallel, real prices reaching an all time low in 1932. Prices then began to recover with increasing production and consumption but half way through the period they began to drop again while production and consumption continued to climb. During the mid 40's fall-off in production and consumption, prices held steady and began to climb with the long period of expansion beginning in 1947. However, production and consumption on the one hand and prices on the other rose together only until 1956; then prices dropped sharply in one year, to remain until 1964 on a plateau which is only slightly above that of the early 20's. In 1965 prices started to climb again, reaching the level of 1956 in 1970, dropped back for a couple of years, and rose again to that level in 1974. The price peaks of 1956, 1970 and 1974 were close to the real price level of the years from 1908 to 1914. Since 1974 real prices have again been dropping, in spite of increasing levels of consumption, to the plateaus of 1920-1928, 1947-1952 and 1957-1964.

The important fact is that significant and sustained price rises did not accompany sustained increases in consumption. This occurred despite the fact that during this whole period there did take place a gradual shift in production from higher grade ores to much lower grade ores. This included major developments in Canada and the U.S.A. (i.e., in economies with relatively high labour costs), reflecting major technological changes.

The history of the copper industry during the last 100 years is complicated by a number of major attempts to interfere with market forces. These were the "Secretan Corner" of 1887, the attempts in 1890 by Anaconda/The Amalgamated Copper Company, and in 1926 by Copper Exporter Incorporated, to organize the market and lastly the formation of CIPEC. As the comparison of production, consumption and price patterns shows, all these attempts on the part of traders or of producers to keep copper prices high failed in spite of often significantly rising levels of consumption.

Today the U.S.A. and the U.S.S.R. are the largest consumers as well as producers. The most important net exporters of mined copper are Canada, Chile, Zaire, and Zambia followed by Peru, the Phillipines and, more recently, Poland. The most important net consumers of imported refined copper are Japan, West Germany, the United Kingdon, France, Belgium and China.

Copper is most commonly traded in the form of cathodes or wire bars. Both are of 99.9% purity. The most significant copper uses are due to its superior thermal and electrical conductivity. Most copper is used by the electrical industry, in the form of copper wiring, followed by brass screws and fittings, some industrial heat exchangers and radiators, water pipes and coins.

The copper industry in North America has long been characterized by a considerable degree of vertical integration which allowed a dual domestic pricing system. There is on the one hand a producer price for large long term contracts, and on the other hand the London Metal Exchange and COMEX spot and futures prices. With increasing tonnages of production from state-run Third World enterprises (which are not vertically integrated to the extent U.S. producers are) entering the market, there has been a movement on the part of major producers to tie their producer prices to LME quotations.

One of the most significant facts to keep in mind when considering the copper market is that copper can be readily substituted in its most important uses. In electrical applications, and in radiators, the traditional substitute is aluminum, with silica products coming up quickly as substitutes for many communication purposes. In the construction industry, plastic pipes have made considerable inroads during periods of high copper prices.

Anticipating somewhat the simulations to be carried out in Chapter 10, attention should be drawn to the following features of the copper market:

Comparing high and low growth scenarios, copper appears to be a typical competitive industry with higher levels of consumption and production being generally accompanied by higher prices, and lower levels of consumption being accompanied by lower prices. The upward movement of prices under high growth scenarios seems however, according to past patterns, to be significantly constrained by three major factors: a) an apparent capability on the part of copper producers to quickly increase production during boom periods; b) the ready and increasing availability of substitutes; and c) the significant role played by a rapidly increasing potential scrap supply. In modelling, these constraints to upward mobility of real copper prices will have to be handled as external shocks to the system. Of further interest might be a comparison of the implications of changes in Third World production and in the production levels of the Centrally Planned Economies. While it appears that reductions in production due to political instability, of for instance 5% in African countries, have almost insignificant effects upon world wide levels of production and consumption and upon the price level, an expansion of production in the Centrally Planned Economies of 5%, while not leading to significant changes in consumption, would lead to significantly lower real prices, which would only begin to recover slowly several years after such a shock is administered to the system.

6.2 DATA

In order to perform our estimations, data were required for primary production, secondary production, and consumption of copper by country group, as well as the world price of copper and the related commodity. In this analysis, we use aluminum as the related commodity, since aluminum is the principal substitute for copper. All data were collected from *Metal Statistics*. Following is a brief description of the data.

For primary production we use mine production, expressed in thousand metric tons. The country specific data is displayed in Table 6-1 and plotted in Figures 6-1 and 6-2. World primary production (i.e., the sum of the country group data) is displayed in column 1 of Table 6-4.

Table 6-1
Copper: Primary Production
(thousands of metric tons)

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1947	973.8	535.7	53.5	382.8	60.0	13.3	207.4
1948	975.7	549.6	63.4	420.8	66.5	12.6	231.4
1949	921.9	479.6	82.8	441.9	72.6	13.8	251.3
1950	1064.6	480.3	93.1	519.9	70.2	15.1	276.1
1951	1087.1	504.6	104.5	559.3	74.2	16.9	306.4
1952	1073.6	520.9	124.4	587.2	76.4	20.4	351.0
1953	1070.2	478.3	125.4	639.3	72.9	37.1	370.4
1954	1032.5	477.6	148.0	681.6	71.1	41.5	390.8
1955	1201.6	554.7	153.4	663.1	80.6	48.1	401.1
1956	1323.6	611.5	184.9	738.3	85.1	55.4	460.7
1957	1311.8	619.2	203.9	770.8	94.2	60.3	478.3
1958	1201.5	601.4	202.8	740.6	100.0	76.9	498.0
1959	1106.9	665.5	219.0	936.3	94.4	96.5	523.6
1960	1378.4	794.4	216.1	982.6	94.9	111.2	585.0
1961	1455.3	808.2	224.6	981.4	98.6	97.2	635.1
1962	1536.3	811.5	231.2	965.6	96.6	110.8	679.3
1963	1511.2	854.9	246.1	988.4	89.8	114.8	733.9
1964	1572.8	871.7	240.3	1044.2	93.0	106.3	786.8
1965	1687.0	842.6	250.1	1121.9	86.9	91.8	799.1
1966	1755.6	914.3	263.3	1140.6	85.5	111.3	857.8
1967	1421.9	929.7	277.8	1182.2	90.0	91.8	966.3
1968	1667.3	954.4	306.3	1206.2	98.7	109.6	1013.5
1969	1921.2	971.4	327.4	1277.2	115.7	130.8	1093.3
1970	2170.3	991.8	353.8	1285.8	114.8	157.8	1194.7
1971	2035.4	999.0	385.2	1294.6	136.0	177.3	1307.2
1972	2230.0	1032.3	402.4	1403.5	150.7	309.8	1386.6
1973	2382.4	1036.1	435.6	1479.8	182.6	403.2	1448.2
1974	2270.2	1214.6	455.7	1463.6	170.0	435.4	1485.8
1975	2016.0	1102.9	455.7	1463.6	180.6	391.5	1582.8
1976	2187.5	1327.4	479.3	1474.2	177.9	390.9	1603.9
1977	2123.8	1500.3	506.5	1460.1	182.1	402.3	1648.7

In the case of secondary production, we employed the reported data for scrap recovery, expressed in thousand metric tons. Data were available only for North America, South America, Asia, Europe, and Oceania. These data are presented in Table 6-2. The sums for the available country groups are displayed in column 2 of Table 6-4.

Table 6-2
Copper: Secondary Production
(thousands of metric tons)

Year	North America	South America	Asia	Africa	Europe	Oceania
1947	.	.	.	.	.	.
1948	251.3	0.0	25.1	.	103.2	0.0
1949	220.6	0.0	38.5	.	172.5	0.0
1950	219.4	0.0	47.9	.	190.5	0.0
1951	150.3	0.0	50.2	.	199.3	0.0
1952	139.4	0.0	45.1	.	181.9	0.0
1953	193.0	0.0	27.4	.	208.3	0.0
1954	186.9	0.0	36.7	.	215.3	0.0
1955	218.3	0.0	32.4	.	252.7	0.0
1956	238.3	0.0	33.7	.	236.2	0.0
1957	215.2	0.0	33.3	.	195.6	0.0
1958	210.9	0.0	20.6	.	193.8	0.0
1959	230.9	0.0	39.0	.	226.6	0.0
1960	264.7	0.0	60.9	.	237.9	0.0
1961	239.2	0.0	74.2	.	297.3	14.9
1962	247.4	0.0	58.3	.	330.6	12.0
1963	261.4	0.0	43.1	.	316.8	16.6
1964	303.1	0.0	59.5	.	347.3	20.5
1965	399.2	0.0	65.6	.	403.2	34.5
1966	441.5	7.0	85.9	.	454.7	24.0
1967	386.1	8.0	67.2	.	441.1	29.8
1968	403.3	12.0	71.2	.	552.8	35.2
1969	461.3	15.0	108.0	.	502.0	37.7
1970	450.3	20.0	102.2	.	542.2	41.3
1971	354.3	25.0	103.0	.	442.6	34.8
1972	375.5	26.0	103.3	.	440.6	34.6
1973	434.9	30.0	133.3	.	414.4	33.1
1974	464.0	40.0	127.2	.	481.5	32.0
1975	324.9	34.0	76.9	.	399.0	26.2
1976	349.6	39.1	95.0	.	422.5	28.1
1977	374.5	34.6	85.3	.	413.9	30.6

Consistent consumption data series were available only for refined copper. These data for the individual country groups, in thousand metric tons, are displayed in Table 6-3 and plotted in Figures 6-3 and 6-4. Total world consumption (i.e., the sum of the country group data) is presented in column 3 of Table 6-4.

Table 6-3
Copper: Consumption
(thousands of metric tons)

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1947	1426.6	47.8	63.1	15.3	855.0	30.5	252.0
1948	1388.3	42.5	93.5	26.4	889.2	32.7	308.0
1949	1116.3	62.8	77.7	23.3	906.2	34.5	362.5
1950	1389.2	61.7	100.5	23.3	981.9	33.3	318.1
1951	1407.1	89.5	133.6	25.9	1037.7	37.9	441.3
1952	1460.6	70.5	131.3	25.8	1067.1	36.0	489.8
1953	1451.2	63.2	125.8	21.0	939.1	35.9	499.5
1954	1231.2	92.0	127.8	20.3	1328.1	40.4	519.3
1955	1488.3	81.4	137.4	22.3	1442.2	52.4	531.1
1956	1512.0	58.3	190.8	26.1	1463.5	44.1	581.1
1957	1333.9	77.3	229.2	34.7	1557.8	50.1	622.7
1958	1248.1	100.7	221.6	33.9	1643.8	59.8	649.7
1959	1445.1	93.1	287.9	26.6	1583.8	59.7	782.9
1960	1331.3	90.1	384.4	30.9	1870.7	73.5	847.0
1961	1455.7	95.8	462.4	32.4	1939.3	61.5	863.1
1962	1551.7	93.2	406.2	37.5	1850.8	69.0	900.0
1963	1749.9	101.9	454.7	43.8	1917.0	83.7	976.9
1964	1879.6	158.2	540.9	48.2	2112.0	99.1	1040.4
1965	2054.6	166.9	504.6	44.5	2098.3	102.3	1104.0
1966	2403.4	126.8	536.2	38.0	1940.6	108.0	1159.7
1967	2003.6	109.2	682.9	37.8	1907.4	88.4	1227.5
1968	1936.2	162.6	761.4	39.7	2108.1	104.7	1263.2
1969	2172.3	186.9	893.0	44.8	2267.2	106.4	1303.7
1970	2089.2	182.3	918.2	44.0	2399.1	114.1	1354.7
1971	2053.8	224.0	909.8	58.0	2295.8	117.3	1413.0
1972	2253.7	316.3	1065.1	66.1	2427.6	112.8	1513.2
1973	2469.3	268.4	1334.3	86.7	2578.9	123.8	1618.9
1974	2265.0	332.8	1001.9	93.4	2578.1	123.9	1710.1
1975	1592.6	306.4	953.6	86.3	2308.7	103.7	1805.9
1976	2023.3	367.6	1224.4	81.7	2518.9	113.6	1872.8
1977	2202.1	393.3	1336.8	78.8	2642.5	114.6	1942.0

For the world price of copper, we used the London Metal Exchange (LME) price for electrolytic copper wirebars. While we acknowledge the difficulties introduced by the two-market system that has existed,[1] we feel that this price more accurately reflects a market determined price. The LME price is expressed in pounds Sterling per long ton. We converted this into U.S. cents per pound. Then, since our models are specified in terms of the real price of the mineral, we deflated this nominal price by the U.S. wholesale price index (1967=100). These data are displayed in column 4 of Table 6-4.

The world price of aluminum was defined to be the U.S. price for 99.5% aluminum ingots in cents per pound. Although this variable has been criticized,[2] we feel that it is more appropriate than the alternative measures. It was also deflated by the U.S. wholesale price index (1967=100). These data are presented in column 5 of Table 6-4.

Table 6-4
World Primary and Secondary Productions
World Consumption and Prices

Year	Primary Production	Secondary Production	Consumption	Real Prices	
				Copper	Aluminum
1947	2226.5	.	2690.3	30.7058	18.3007
1948	2320.0	379.6	2780.6	29.1160	17.7935
1949	2263.9	431.6	2583.3	21.1308	20.3304
1950	2519.3	457.8	2908.0	27.3233	20.4315
1951	2653.0	399.8	3173.0	30.0462	19.7585
1952	2753.9	366.4	3281.1	36.7206	20.7788
1953	2793.6	428.7	3135.7	33.4783	22.5458
1954	2843.1	438.9	3359.1	35.2822	23.0639
1955	3102.6	503.4	3755.1	50.0990	24.9260
1956	3459.5	508.2	3875.9	45.0982	26.4961
1957	3538.5	444.1	3905.7	30.0305	27.2412
1958	3421.2	425.3	3957.6	26.1046	26.2051
1959	3642.2	496.5	4279.1	31.3503	26.0949
1960	4162.6	563.5	4627.9	32.4411	27.3973
1961	4300.4	625.6	4910.2	30.4543	26.9397
1962	4431.3	648.3	4908.4	30.8801	25.1846
1963	4539.1	637.9	5327.9	30.9537	23.9397
1964	4715.1	730.4	5878.4	46.1738	25.0697
1965	4879.4	902.5	6075.2	60.6216	25.3696
1966	5128.4	1013.1	6312.7	69.2653	24.5491
1967	4959.7	932.2	6056.8	44.9188	24.9780
1968	5356.0	1074.5	6375.9	54.5526	24.9590
1969	5837.0	1124.0	6974.3	62.4649	25.5174
1970	6269.0	1156.0	7101.6	57.0581	26.0109
1971	6334.7	959.7	7071.7	44.4559	25.4609
1972	6915.3	980.0	7754.8	37.6546	22.1738
1973	7367.9	1045.7	8480.3	55.9842	18.5598
1974	7495.3	1144.7	8105.2	57.4728	21.3198
1975	7193.1	861.0	7157.2	28.7455	22.7479
1976	7641.1	934.3	8202.3	32.4167	24.2301
1977	7823.8	938.9	8710.1	.	.

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[1] See Banks (1974) and Fisher, Cootner, and Baily (1972).

[2] See Fisher, Cootner, and Baily (1972).

FIGURE 6-1
COPPER PRIMARY PRODUCTION
NORTH AMERICA (▲), SOUTH AMERICA (■)
AFRICA (●), CENTRALLY PLANNED (△)
(thousands of metric tons)

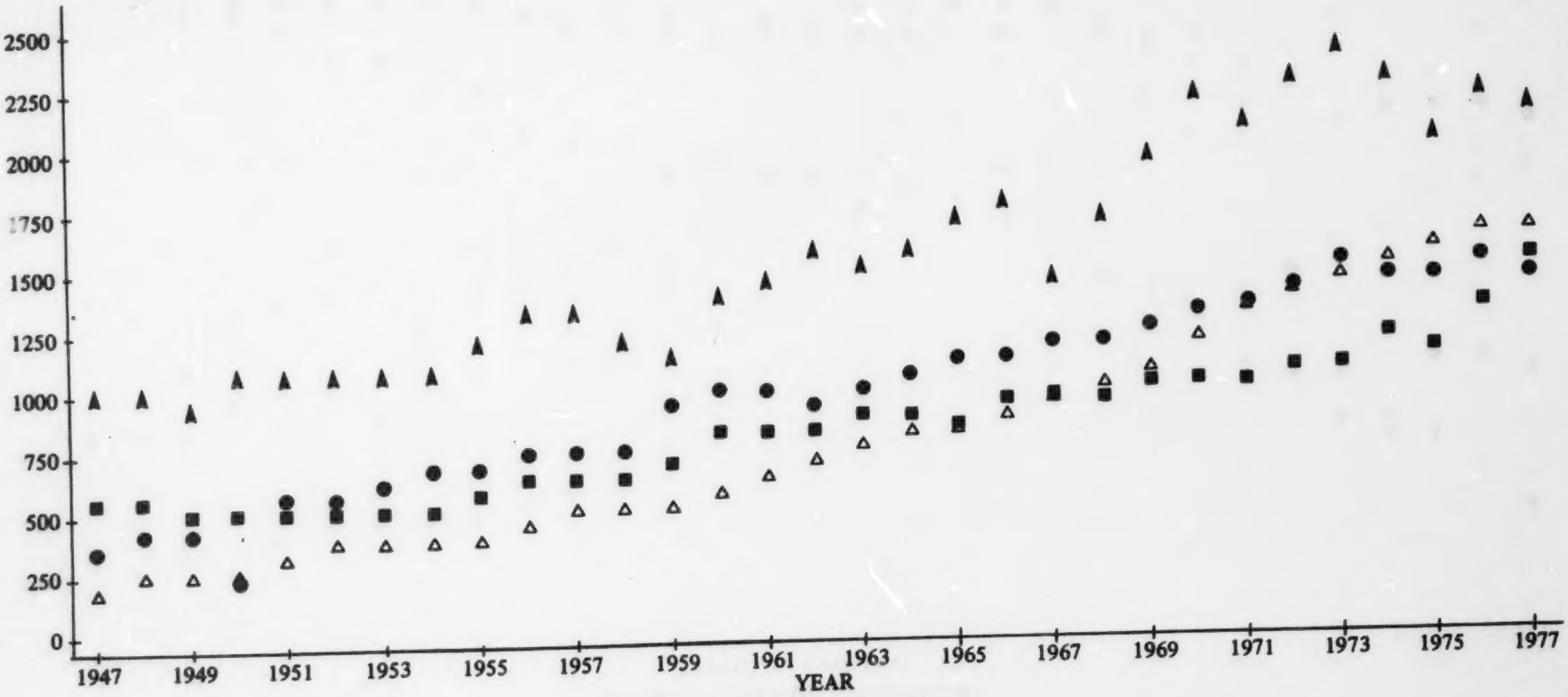


FIGURE 6-2
COPPER PRIMARY PRODUCTION
ASIA(▲), EUROPE (■), OCEANIA (●)
(thousands of metric tons)

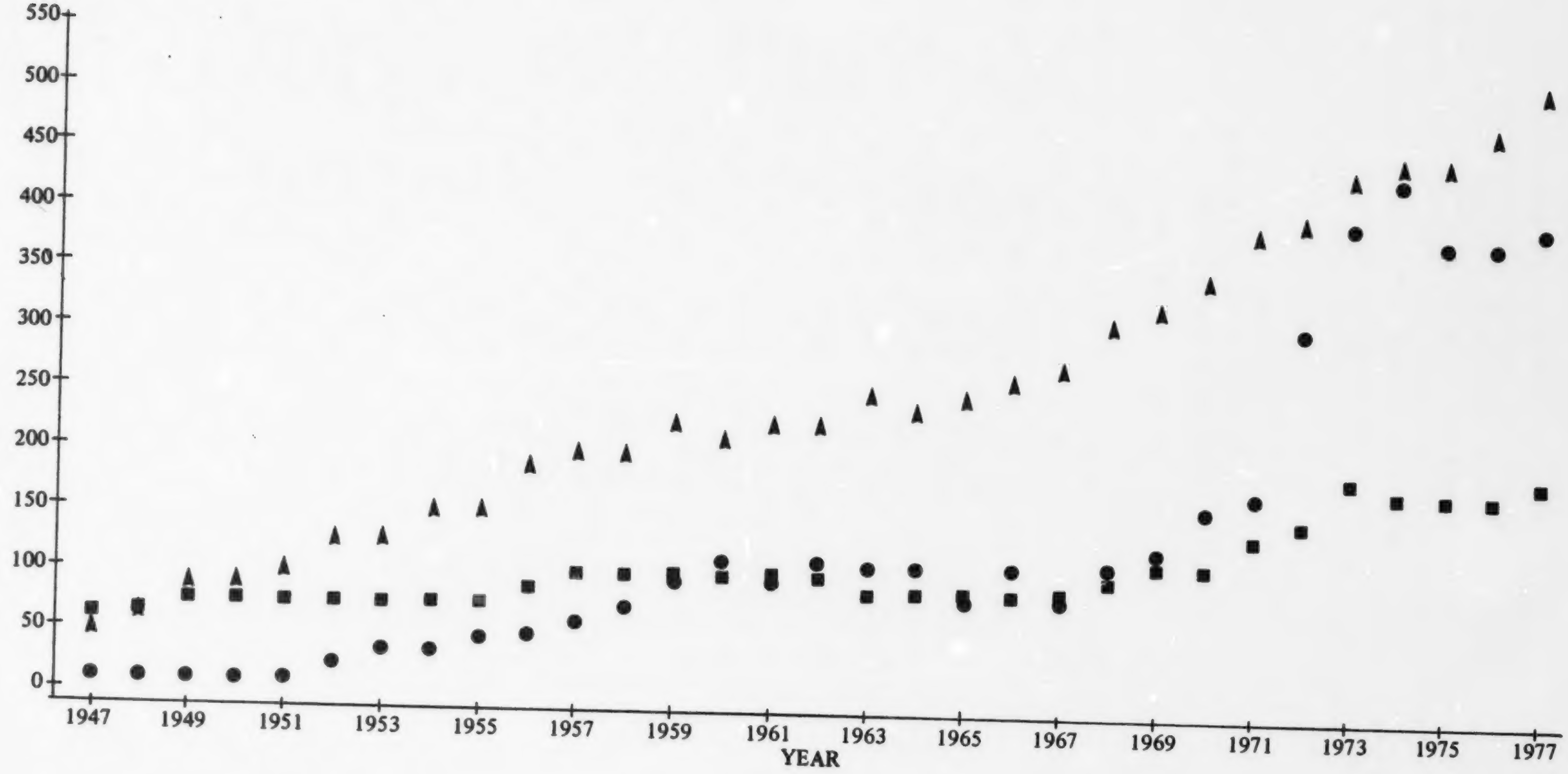


FIGURE 6-3
COPPER CONSUMPTION
NORTH AMERICA (▲), EUROPE (■), CENTRALLY PLANNED (●)
(thousands of metric tons)

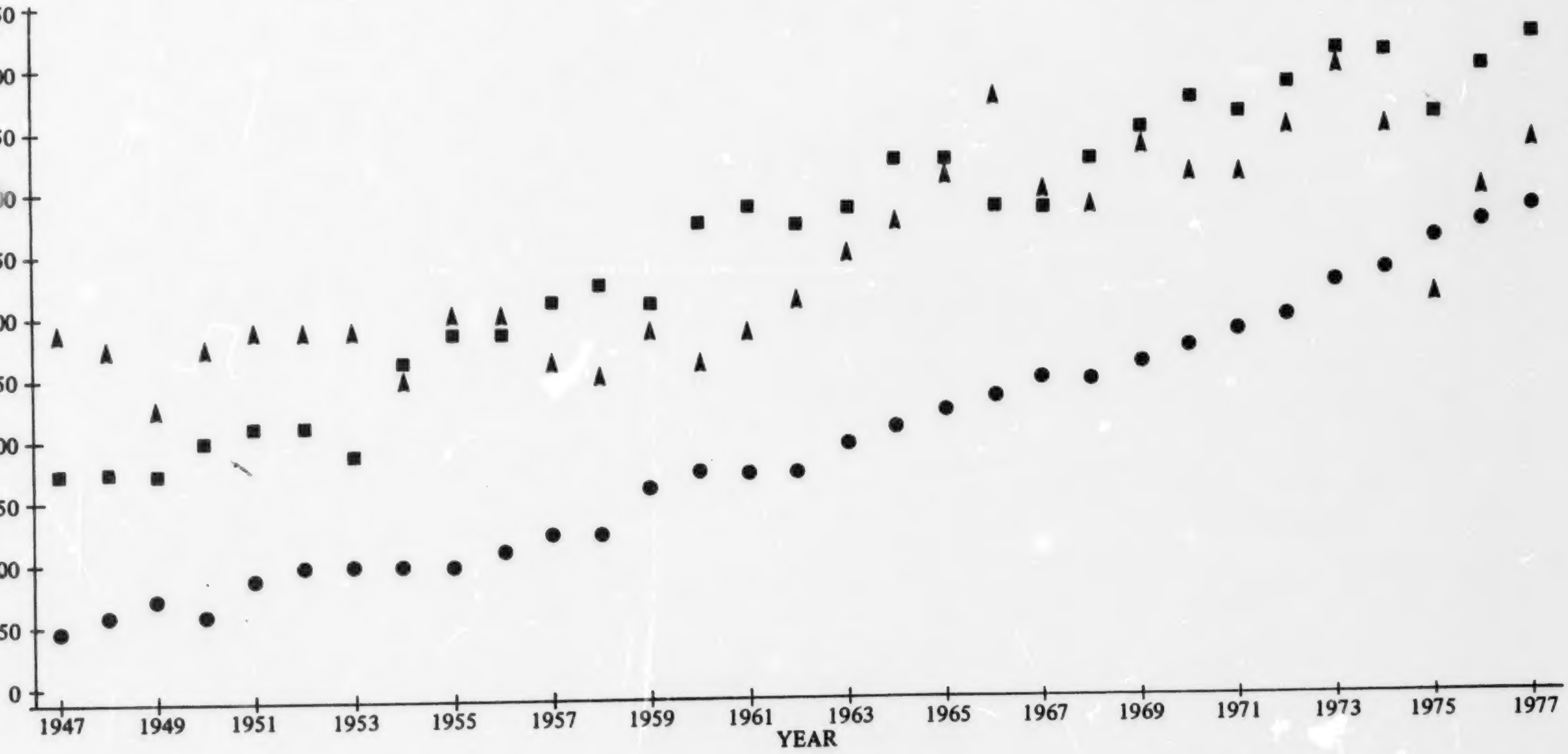
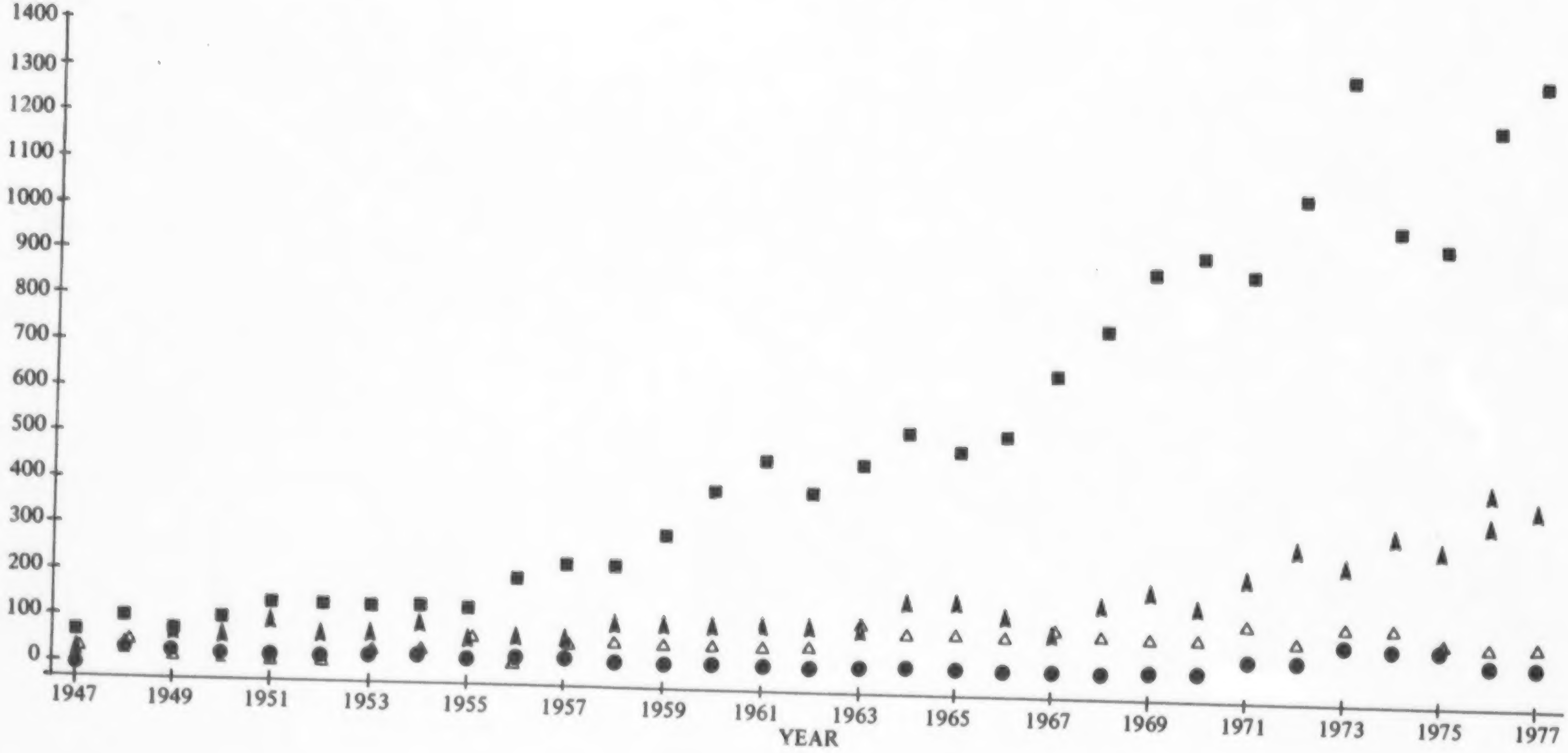


FIGURE 6-4
COPPER CONSUMPTION
SOUTH AMERICA (▲), ASIA (■)
AFRICA (●), OCEANIA (△)
(thousands of metric tons)



6.3 EMPIRICAL RESULTS

Using the data described in Chapter 5 and in the preceding section, we estimated the econometric equations developed in Chapter 3 (and reviewed in Chapter 5). The results of these estimations are presented in Tables 6-5 through 6-8. Following is a brief discussion of these estimates.

TABLE 6-5
ESTIMATED EQUATIONS
PRIMARY PRODUCTION
COPPER
(T-STATISTICS IN PARENTHESES)

		North America 1952-1976	South America 1952-1976	Asia 1952-1976	Africa 1952-1976	Europe 1952-1976	Oceania 1952-1976	Centrally Planned 1952-1976
θ_0	Constant	300.8920 (0.1782)	-1587.09 (-1.5241)	85.4435 (0.3782)	-1063.49 (-1.5440)	-700.2360 (-1.6793)	472.2330 (1.1998)	-344.858 (-1.1844)
θ_1	$P_{M,t}$		1.6620 (0.5563)	0.4449 (0.5015)		0.9507 (1.1922)	1.6589 (1.1224)	
θ_2	$P_{M,t-1}$	6.2525 (1.5930)	-0.8037 (-0.5152)	0.7336 (1.8391)		-0.1077 (-0.2646)	-1.8313 (-1.4660)	1.6715 (1.9922)
θ_3	$ROWS_{M,t-1}$	0.6708 (1.9057)	0.0852 (0.5210)	0.0419 (0.9866)	-0.0138 (-0.2406)	0.0660 (2.0815)	0.1182 (1.4830)	0.0425 (0.8403)
θ_4	$D_{M,t}$	0.5061 (4.5440)		0.0231 (1.4478)	0.0521 (1.5712)	0.0094 (0.7452)		0.0007 (0.0218)
θ_5	$D_{M,t-1}$	-0.2436 (-1.5118)	-0.0887 (-0.6781)	-0.0433 (-1.4718)	0.0079 (0.2248)	-0.0452 (-2.1050)	-0.0026 (-0.0360)	-0.0288 (-0.7224)
θ_6	t	-30.1440 (-0.8850)	39.0058 (1.9299)	-2.9154 (0.6766)	27.2426 (1.8609)	12.0909 (1.5370)	-14.8840 (-1.5772)	5.4933 (0.8398)
θ_7	r_t	26.5248 (0.4086)	NA	NA	NA	-5.6891 (-0.5683)	-26.5812 (-1.0092)	NA
θ_8	r_{t-1}	101.9810 (1.3034)	NA	NA	NA	13.8761 (1.6991)	34.0857 (0.9765)	NA
θ_9	$\Sigma^{t-2} Q^S_{M,i}$	-0.0810 (-2.1560)	-0.0137 (0.4345)	-0.0030 (-0.1506)	-0.0056 (-0.3925)	-0.1863 (-2.0840)	-0.0283 (-0.4520)	-0.0105 (-0.6256)
θ_{10}	$Q^S_{M,t-1}$	0.2280 (1.0068)	0.2261 (0.5443)	0.9707 (4.5329)	0.1581 (0.6615)	0.7826 (1.7746)	0.7702 (2.6270)	1.0578 (4.0941)
	R^2 SEE	0.9676 96.7742	0.9569 56.3958	0.9922 11.6735	0.9857 40.0443	0.9566 9.6135	0.9620 31.2156	0.9976 24.4527

TABLE 6-6
ESTIMATED EQUATIONS
SECONDARY PRODUCTION
COPPER
(T-STATISTICS IN PARENTHESES)

		North America 1952-1976	South America 1967-1976	Asia 1952-1976	Europe 1952-1976	Oceania 1962-1976
ϕ_0	Constant	-42.9657 (-1.1154)	-15.7970 (-1.4475)	0.1277 (0.0137)	-70.9817 (-1.3774)	-19.0611 (-1.5382)
ϕ_1^P	$P_{M,t}$	2.5640 (3.1316)	0.2370 (1.6815)	0.5298 (1.8702)	3.0077 (3.1027)	0.2158 (0.8958)
ϕ_2^P	$P_{M,t-1}$	-1.6662 (-1.6742)	-0.0086 (-0.0619)	-0.1909 (-0.6764)	-0.3807 (0.3178)	-0.0550 (-0.1633)
ϕ_3^P	$P_{M,t-2}$					0.3801 (1.5400)
ϕ_1^D	$Q_{M,t}^D$	0.0689 (1.8272)	0.0442 (0.8982)	0.0631 (3.2277)	0.0474 (0.6593)	0.2281 (0.6359)
ϕ_2^D	$Q_{M,t-1}^D$	0.0112 (0.2595)	-0.0220 (-0.4217)	0.0337 (0.9493)	-0.0593 (-0.7494)	0.1949 (0.5166)
ϕ_3^D	$Q_{M,t-2}^D$	-0.0163 (-0.5201)	0.0498 (2.3572)	-0.0483 (-2.0235)	0.0790 (1.1795)	-0.2247 (-0.9896)
ϕ_4^D	$Q_{M,t-3}^D$					0.0138 (0.0854)
ϕ_S	$Q_{M,t-1}^{SCRAP}$	0.6666 (3.9492)	0.6242 (0.9779)	0.3493 (1.5652)	0.5421 (2.9635)	
	R^2	0.9476	0.9852	0.9298	0.9148	0.7639
	SEE	26.4136	2.3446	9.8551	40.0227	5.6127

TABLE 6-7
ESTIMATED EQUATIONS
DEMAND
COPPER
(T-STATISTICS IN PARENTHESES)

		North America 1952-1976	South America 1962-1976	Asia 1962-1976	Africa 1962-1976	Europe 1952-1976	Oceania 1952-1976	Centrally Planned 1952-1976
Ψ_0	Constant	-114.826	-518.065	-291.800	-114.792	-106.671	-67.2024	6.4491
		(-0.4842)	(-2.7784)	(-0.5656)	(-3.9083)	(-0.7070)	(-2.6201)	(0.4069)
Ψ_1	$P_{M,t}-P_{R,t}$		-0.5497			-2.7735	-0.1706	
			(-0.3868)			(-0.7823)	(-0.6209)	
Ψ_2	$P_{M,t-1}-P_{R,t-1}$	-3.1211*	-2.1104	-0.9659	-0.5824	-8.1560	-0.3774	-1.0715
		(-0.9082)	(-1.5697)	(-0.1882)	(-3.1071)	(-2.5275)	(-1.8155)	(-1.7999)
Ψ_3	$S_{M,t-1}-D_{M,t-1}$	-0.3728	-0.0694	-0.0452	-0.0122	-0.1867	-0.0246	-0.0490
		(-2.1467)	(-0.8837)	(-0.1914)	(-1.5672)	(-1.1391)	(-1.9769)	(-2.0045)
Ψ_4	Y_t	5223.39	649.865	4003.75	13.8973	3194.36	290.696	
		(4.1918)	(0.9597)	(2.2777)	(0.2035)	(2.0486)	(2.8854)	
Ψ_5	Y_{t-1}	-3424.65	112.119	-3300.86	146.270	-1089.49	-157.902	509.117
		(-2.2090)	(0.1457)	(-1.7130)	(1.7110)	(-0.6042)	(-1.3612)	(2.7489)
Ψ_6	$\dot{K}_{t-1}$	831.093	-80.1161	1154.48	-131.461	2029.80	96.4749	NA
		(0.9988)	(-0.3312)	(1.1363)	(-2.8752)	(2.6625)	(1.9636)	
Ψ_7	$\dot{M}_{t-1}$	5.9988	NA	NA	NA	-9.2975	-0.0260	NA
		(0.2890)				(-0.8232)	(-0.0770)	
Ψ_8	$Q^D_{M,t-1}$	-0.0059	-0.1371	0.1589	0.6064	0.1692	0.2711	0.6749
		(-0.0187)	(-0.3743)	(0.3011)	(4.3208)	(0.8387)	(1.0945)	(4.9975)
	R^2	0.9169	0.9482	0.9124	0.9649	0.9694	0.9694	0.9947
	SEE	131.219	29.1006	112.048	5.0854	98.1609	6.4320	24.2376

*Estimated coefficient for $P_{M,t-2} - P_{R,t-2}$

TABLE 6-8
ESTIMATED EQUATIONS
PRICE
COPPER
(T-STATISTICS IN PARENTHESES)
1952-1976

η_0	Constant	54.2536 (0.2216)
η_1	$P_{M,t-1}$	-0.1445 (-0.4358)
η_2	$P_{M,t-2}$	-0.4109 (-1.8680)
η_3	$S_{M,t-1} - D_{M,t-1}$	-0.0273 (-1.8020)
η_4	t	-4.4307 (-0.8778)
η_5	r_t	-3.5678 (-0.8211)
η_6	$\Sigma^{t-2} S_{M,i}$	0.0002 (0.1805)
η_7	$WQ^{SCRAP}_{M,t-1}$	0.0142 (0.4990)
η_8	$P_{R,t}$	2.5548 (1.8904)
η_9	$P_{R,t-1}$	0.0172 (0.0097)
η_{10}	$P_{R,t-2}$	–
η_{11}	Y_t	234.666 (3.0319)
	R^2	0.8396
	SEE	6.6513

6.3.a PRIMARY PRODUCTION

In the case of North America, current price was excluded from the estimation. While this exclusion might be interpreted as a reflection of the inability of producers to change their production schedule in response to unanticipated price changes, it may instead be the result of the two-market system that existed over most of this period and the price setting behavior of the North American producers. It is significant to note that, with the exclusion of current price, the sign of lagged price is positive as was predicted. In this case, the sign on lagged production in the rest of the world ($ROWS_{M,t-1}$) is, however, opposite of that predicted. The remainder of the signs are as predicted. Of particular interest are the very significant effect of current world consumption and the insignificant effect of the interest rate. Both of these may well be the result of the price setting behavior of the North American producers.

For South America, our estimations indicated the exclusion of current world consumption as a determinant. The signs of the estimated coefficients are all as would be predicted. Examination of these results would indicate that the primary determinant of South American production is trend (i.e., t).

The estimates for Asia all have the predicted signs. However, consideration of the levels of significance tends to indicate the existence of a strong trend effect (i.e., $Q^S_{M,t-1}$) modified by price and consumption conditions.

In Africa, neither of the price terms proved to be satisfactory; so, both were excluded. In such a context the sign of $ROWS_{M,t-1}$ remains indeterminant and is, in this case, insignificant. Indeed, these estimates would indicate a strong time trend in production modified by changes in the world consumption of copper.

The estimates for European production all carried the expected sign. It would appear that both price and world consumption have substantial impacts on primary production from this country group. It is also interesting to note the level of significance for $\sum^{t-2} Q^S_{M,i}$. As has been shown, this term reflects the effect of the declining stock of available ore. As might be anticipated, this effect is pronounced in Europe.

In Oceania, current world consumption was eliminated as a determinant. Overall, these estimates tend to indicate that expected demand is not a significant determinant of production in Oceania. This surmise is supported by the existence of a strong trend relation (i.e., $Q^S_{M,t-1}$).

Current price proved unsatisfactory in the primary production function for the Centrally Planned Economies. This conclusion, indicating a lack of response to changes in price, is supported by the fact that current world consumption has an insignificant effect and the existence of an extremely significant trend effect, as demonstrated by the coefficient of $Q^S_{M,t-1}$. In this case, the coefficient for lagged production in the rest of the world carries a sign opposite that which would be predicted theoretically; but, the estimate is not statistically significant.

6.3.b SECONDARY PRODUCTION

As has been noted previously, scrap data was available for only five of the seven country groups. The estimates of the scrap supply function as described in Chapters 3 and 5 are presented in Table 6-6.

As can be noted, the most unusual results were obtained for Oceania. We, therefore, will consider these first. The signs of the estimated coefficients are as predicted except that lagged scrap production is excluded. This exclusion implies that the lag weights are relatively small (i.e., current scrap production is not sensitive to prior consumption in Oceania). This supposition is supported by the fact that the data series for scrap is extremely short and that the price variables are most significant. It would appear then that scrap production in Oceania is in an early stage and therefore reflects primarily the impact of new rather than old scrap.

In the remainder of the country groups, the signs of the estimated coefficients were as predicted. While the current price variables were significant for all of these country groups, they were apparently most significant in North America and Europe. The estimated parameters for consumption in the country groups were not as clear but it would appear that scrap production is more heavily influenced by current consumption (i.e., new scrap) in North America and Asia and by prior consumption (i.e., old scrap) in South America and Europe.

6.3.c DEMAND

In this set of estimates the results for North America were most unusual. As was the case with primary production, the current price variable proved to be unsatisfactory and was excluded. However, in the demand function, the once lagged price term also proved unsatisfactory. We would posit that this results from the fact that we use the LME rather than the EMJ price. Given the previously mentioned two-market system in copper, it is quite likely that the controlled EMJ price is influenced by the market-determined LME price. If LME price rises in period t, then we might expect an increase in the EMJ price in succeeding periods —

$$\mathrm{EMJ}_t = f(\mathrm{LME}_{t-1}).$$

Using this assumption, we then included the twice lagged price variable. The estimated negative relation would tend to support our contention of the lagged relation between the two prices.[3] Since we excluded current price effects, we would have expected the sign of $S_{M,t-1} - D_{M,t-1}$ to be positive. The estimated negative sign could result from the existence of the two-market system or simply the high level of correlation among the variables. The only other troublesome sign is that for lagged consumption; as indicated by the t-statistic, this estimate is not significantly different from zero.

For South America, only the sign of lagged consumption is other than as predicted, but as in the case of North America, the estimate is statistically insignificant. It is interesting to note that the price effect is extremely significant in this country group.

The current price effect was excluded in both Asia and Africa. As was described in Chapter 3, this indicates that there is virtually no adjustment to unanticipated price conditions. (In both cases, the sign of $S_{M,t-1} - D_{M,t-1}$ is opposite of that predicted.) This lack of adjustment may well be explained by noting that, in Asia, demand is most significantly influenced by income while the demand in Africa is strongly influenced by trend $(Q^D_{M,t-1})$.

All of the estimated coefficients for Europe and Oceania carried the expected signs. Indeed, examination of these estimates and their levels of significance would tend to indicate that only the rate of growth of the money supply would be a questionable determinant of demand.

In the case of the Centrally Planned Economies, our estimates indicated the lack of adjustment to both unanticipated price and income (i.e., both current prices and current income are excluded). This finding is supported by the level of significance of the trend term $(Q^D_{M,t-1})$, indicating that there may exist long-run consumption decisions. Again, after the exclusion of the current price variables, the sign of $S_{M,t-1} - D_{M,t-1}$ is opposite to that which would be predicted.

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[3] Some additional support for this contention would be derived from the fact that, in the earlier version of this analysis [Smithson, *et alia* (1979)], an inverse relation was estimated between quantity demanded and once lagged EMJ price.

6.3.d MARKET PRICE

Our estimated price equation is presented in Table 6-8. As we have noted previously, we make no hypotheses concerning the signs of the parameters. There are, however, several features of particular interest. First, while the impact of once lagged price is insignificant, the effect of twice lagged price is significant and negative. As might be expected, our results indicate that an increase in inventories (reflected by $S_{M,t-1} - D_{M,t-1}$) would tend to decrease price; and this effect is statistically significant. The price of aluminum also exerts a statistically significant effect on the price of copper. An increase in the price of aluminum would tend to increase the price of copper. Finally, per capita income [in North America] exerts a significant, positive influence on price. Our estimates indicate that an increase in income would tend to increase the price of copper.

- 7 -

ALUMINUM

7.1 AN OVERVIEW OF THE ALUMINUM MARKET

While neither aluminum nor bauxite is produced in Ontario the aluminum markets were treated in this project because aluminum is an important substitute for copper. The most notable fact is that while production and consumption of aluminum have increased over 10-fold since World War II, real prices today move in the same range as they were 35 years ago — which is one half their 1920's - 1930's level or one eighth of their turn of the century level. There have, of course, been some fluctuations in the short run.

The aluminum industry has been characterized by a significant degree of vertical integration. It is dominated essentially by three major firms, Alcoa/Alcan, Reynolds and Kaiser. In recent years many producers of aluminum have been affected by increases in energy costs and increases in bauxite mining taxes in many of the Third World countries which produce large amounts of bauxite.

Aluminum was introduced to the public at the 1855 Paris Exhibition. It was originally a curiosity priced higher than silver. It was only after the discovery of electrolytic methods of aluminum production by Hall and Heroult that production increased significantly.

In the last decade of the last century the price of aluminum was in real terms approximately twice as high as that of copper, and ten times as high as it is now. The price steadily dropped to reach one fifth of its 1895 price or double its current price in real terms at the end of World War I. However this drop was not steady. It was interrupted by sharp peaks in 1907 and 1916. In considering aluminum prices it is revealing to view their movements in relation to copper prices. After real prices for aluminum had approached those of copper at the end of World War I, the dominant pattern was one in which aluminum prices fell when copper prices rose and vice versa. This pattern was only interrupted during the World War II period, when both prices declined slightly in real terms and roughly in parallel, and for a few years during the 1950s. Then both prices rose, but those for copper rose far more rapidly than those for aluminum. Since 1960 aluminum prices have been stable most of the time, except during a dip in the early 70s corresponding to a peak in copper prices.

The use of aluminum as a mass consumption metal really began after 1932 when copper prices rose steeply while aluminum prices fell rapidly. Considering the generally opposite price movements of aluminum and copper and looking at the movements of aluminum and copper consumption one may suggest that rising copper prices typically led to new forms of substitution of aluminum for copper — substitutions which were irreversible during periods of declining copper and rising aluminum prices. This may be explained in part by the much greater stability in real terms of aluminum prices and in part by the fact that copper is, depending on the alloys considered, between 3.2 times and 3.5 times heavier than aluminum. Thus in most uses a unit of weight of aluminum will go 3 times as far as the same weight of copper. After the immediate post World War II adjustments the sharp drop in real copper prices from 1948 to 1949 saw a continued rise in the consumption of aluminum. The sharp drop in copper prices from 1956 to 1958 was accompanied by only a very small reduction in aluminum consumption from 1956 to 1957. The weak but stable copper prices from 1960 to 1965 were associated with steep increases in the consumption of aluminum. The same happened during the periods of steep decreases in the price of copper from 1970 to 1972 and the weakening of copper prices from 1976 onward. Only the 1975 recession was characterized by a drop in aluminum consumption.

Significant substitutes for aluminum, other than copper, are magnesium and titanium for structural uses, particularly in the transportation and aircraft industries, but usually at higher unit costs. In the container market plastics, paper, glass and steel are important actual or potential substitutes and steel, wood and laminated wood are substitutes in construction uses where weight and cost are of secondary importance.

Looking ahead at the simulations to be performed in Chapter 10, it is apparent that any external shocks to the system, such as changes in growth levels in any part of the world or changes in production in the Centrally Planned Economies or in the underdeveloped parts of the world, will have insignificant effects upon aluminum consumption, production and prices world wide. Such changes in patterns would lead to differences from the base simulation to only insignificant extent. The important factors are changes in overall world growth rates, and here the aluminum market is going to behave quite differently from that of copper. While a high growth scenario would lead to significant increase in production and consumption it would be accompanied by considerable reductions in aluminum prices. The extent of price decline shown in the simulation would of course not be realized. On the other hand a low growth scenario with significant decreases in production and consumption would be accompanied by increases, even a doubling, in the real prices of aluminum through the 1980s. In other words the past pattern of behaviour of aluminum prices would continue: aluminum prices going down when copper prices are going up and aluminum prices going up when consumption is going down, suggesting a continuation of the past price leadership structure in the aluminum industry. Interest in substituting raw materials other than bauxite, particularly raw materials readily available in North America, such as clays, oil shales and others is going to continue to increase!

7.2 DATA

For the simulations, data were necessary for primary production, secondary production, and consumption of aluminum by country group, as well as the world price of aluminum and the related commodity. We will use, in our analysis, copper as the related commodity. As noted in Chapter 6, aluminum and copper are normally considered to be substitutes in the production of final commodities. All of the data were collected from *Mineral Statistics*. Following is a brief description of the data.

As primary production, we employ the data reported for production of primary aluminum expressed in thousand metric tons. The country specific data are presented in Table 7-1 and plotted in Figures 7-1 through 7-3. World primary production (i.e., the sum of the country group data) is presented in column 1 of Table 7-4.

Table 7-1
Aluminum: Primary Production
(thousands of metric tons)

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1947	790.0	0.0	6.0	0.0	157.3	0.0	96.5
1948	898.6	0.0	12.9	0.0	201.4	0.0	111.1
1949	882.7	0.0	26.0	0.0	215.2	0.0	131.7
1950	1011.9	0.0	30.3	0.0	243.8	0.0	219.9
1951	1164.8	0.4	43.7	0.0	358.8	0.0	239.9
1952	1303.7	1.1	50.2	0.0	414.9	0.0	262.4
1953	1630.9	1.2	54.2	0.0	446.6	0.0	321.1
1954	1833.8	1.5	65.2	0.0	489.1	0.0	430.8
1955	1971.7	1.7	72.0	0.0	532.3	1.3	525.7
1956	2085.8	6.3	81.4	0.0	597.1	9.3	557.7
1957	2000.2	8.9	84.1	7.6	622.4	10.8	653.2
1958	1999.3	11.9	101.6	31.9	636.7	11.1	734.2
1959	2316.4	18.1	125.0	42.3	705.5	11.6	802.7
1960	2518.8	16.5	159.7	43.9	834.8	11.8	890.7
1961	2328.6	18.6	181.1	45.6	903.7	13.4	1026.1
1962	2543.7	35.2	217.6	52.2	978.2	16.5	1078.8
1963	2750.4	26.4	291.0	52.9	1054.8	41.9	1102.5
1964	3080.4	44.4	341.4	51.5	1167.9	80.0	1211.4
1965	3242.2	63.1	376.5	50.5	1240.8	87.8	1439.7
1966	3500.2	75.5	436.0	48.2	1398.3	92.0	1568.5
1967	3839.7	84.7	491.1	98.8	1508.1	92.8	1727.9
1968	3841.2	117.9	618.5	155.9	1703.5	97.4	2004.4
1969	4419.6	141.6	726.2	159.8	1816.6	126.4	1939.1
1970	4569.6	167.4	932.8	165.4	1967.4	205.6	2288.4
1971	4536.0	224.1	1119.7	191.2	2257.0	246.0	2217.0
1972	4646.9	209.4	1304.7	231.9	2445.4	293.5	2362.2
1973	5050.3	231.0	1439.1	249.1	2765.9	323.3	2658.6
1974	5454.9	252.0	1463.3	279.0	3154.0	329.4	2714.9
1975	4397.2	270.1	1410.0	273.1	3065.4	322.8	2798.9
1976	4487.7	317.2	1362.7	337.2	3146.3	372.1	2874.2
1977	5093.9	359.5	1608.8	368.2	3292.8	392.7	2884.8

For secondary production we employ the data for scrap recovery. In the case of aluminum, data were available for all of the country groups except the Centrally Planned Economies. These data, in thousand metric tons are presented in Table 7-2. Aggregating over the available country groups, total world secondary production is presented in column 2 of Table 7-4.

Table 7-2
Aluminum: Secondary Production
(thousands of metric tons)

Year	North America	South America	Asia	Africa	Europe	Oceania
1947	•	•	•	•	•	•
1948	264.4	1.0	4.4	0.0	172.6	0.3
1949	167.8	1.0	3.4	0.0	162.5	0.3
1950	226.1	1.0	5.6	0.4	196.3	0.4
1951	272.9	1.0	6.8	0.5	193.5	0.4
1952	283.8	1.0	10.1	0.5	187.5	0.4
1953	341.3	1.0	12.6	0.5	202.5	0.4
1954	270.1	1.5	15.4	0.8	234.8	0.4
1955	311.7	1.1	15.1	1.0	281.8	0.4
1956	314.7	1.6	18.1	1.0	275.3	0.5
1957	335.1	2.2	23.7	1.2	286.2	0.4
1958	269.4	2.1	28.4	2.1	313.2	3.4
1959	333.5	2.5	43.0	2.7	341.7	4.2
1960	307.2	3.0	57.0	2.7	379.5	4.5
1961	453.7	4.5	78.8	2.7	387.4	4.8
1962	543.8	4.5	81.5	3.0	424.1	5.0
1963	614.6	4.5	104.5	3.0	473.5	5.0
1964	665.7	5.0	119.8	3.0	526.9	7.5
1965	795.7	6.1	138.7	5.0	556.6	8.0
1966	860.5	17.6	158.2	6.5	601.8	12.8
1967	852.7	17.8	202.1	6.5	606.1	14.4
1968	967.3	20.2	248.2	7.0	686.2	17.4
1969	1099.1	26.1	301.4	7.0	802.9	19.3
1970	968.9	26.6	344.2	7.0	814.0	19.9
1971	1037.5	28.9	373.4	8.0	825.5	21.9
1972	1053.3	41.0	436.9	8.5	886.6	21.5
1973	1156.7	46.7	571.1	8.5	994.7	30.6
1974	1192.9	55.7	553.9	9.5	1031.2	38.4
1975	1150.3	49.4	465.3	10.0	887.3	25.8
1976	1351.8	51.6	561.6	9.8	1075.2	28.8
1977	1376.7	48.7	615.5	11.0	1162.9	30.0

The consumption of aluminum by country group is displayed in Table 7-3. These data are expressed in thousand metric tons. The plots of these data are shown in Figures 7-4 through 7-6. Total world consumption of aluminum (i.e., the sum of the country group data) is presented in column 3 of Table 7-4.

Table 7-3
Aluminum: Consumption
(thousands of metric tons)

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1947	609.9	13.1	10.7	1.0	346.0	4.5	108.6
1948	718.4	12.6	13.7	1.0	363.3	5.0	132.1
1949	649.1	14.4	19.5	1.0	364.4	7.6	152.7
1950	882.1	21.9	27.6	2.0	396.8	6.3	244.9
1951	955.3	22.2	41.4	2.5	517.3	11.8	257.1
1952	1048.1	12.5	41.1	3.0	568.2	8.5	273.4
1953	1457.9	21.1	48.4	3.5	518.0	8.1	329.8
1954	1332.9	37.3	60.6	5.0	648.2	11.2	445.1
1955	1664.8	30.2	70.6	6.5	806.6	20.0	505.7
1956	1692.7	43.0	83.8	6.1	842.6	20.0	533.1
1957	1385.1	45.4	98.0	8.5	802.3	18.9	620.1
1958	1426.4	56.9	109.3	9.2	835.6	26.8	651.7
1959	1925.8	52.0	151.4	10.3	1012.6	33.4	776.6
1960	1641.9	62.5	198.4	14.0	1240.1	39.4	891.1
1961	1900.4	85.4	239.6	15.4	1155.9	31.9	998.3
1962	2196.2	107.0	250.4	18.7	1216.2	47.0	1045.0
1963	2478.1	83.2	319.8	21.3	1319.9	54.0	1081.5
1964	2691.3	96.8	370.4	27.4	1486.7	72.0	1154.6
1965	3020.9	126.3	407.1	27.9	1510.8	75.7	1351.6
1966	3489.3	130.0	556.5	43.4	1744.8	83.8	1450.9
1967	3309.3	153.3	700.5	35.8	1733.1	92.6	1622.1
1968	3790.0	167.3	851.6	45.2	2008.6	110.8	1730.8
1969	3917.8	194.5	1034.4	61.6	2343.9	116.8	1798.1
1970	3708.3	201.3	1211.0	70.5	2506.3	137.6	1961.2
1971	4185.5	240.4	1307.2	91.3	2421.7	153.6	2169.4
1972	4577.8	305.0	1546.8	91.4	2703.1	127.3	2173.2
1973	5408.5	376.6	1968.7	107.9	3092.0	175.1	2284.1
1974	5487.4	396.7	1684.5	107.2	3239.3	202.3	2432.5
1975	3558.3	403.9	1591.9	111.3	2677.9	153.3	2482.7
1976	4812.7	411.7	2084.0	118.0	3350.9	186.5	2642.0
1977	5088.4	432.2	1894.5	127.5	3351.8	198.6	2759.6

The world price of aluminum was defined to be the U.S. price for 99.5% aluminum ingots in cents per pound. We realize that the U.S. price may have some characteristics of an administered price[1], but, we feel that the variable is more appropriate than the London or German prices. To express these values in real terms, we deflated the nominal values by the U.S. wholesale price index (1967=100). These values are displayed in column 4 of Table 7-4.

Finally, the world price of copper was defined to be the London Metal Exchange (LME) price for electrolyte copper wirebars. After reviewing the discussions of the two-market system for copper[2], we felt that this is the most appropriate variable to reflect a market price for copper. Since this variable is reported in pounds Sterling per long ton, we converted it to U.S. cents per pound and deflated it with the U.S. wholesale price index (1967=100).

Table 7-4
Aluminum
World Primary and Secondary Production
World Consumption and Prices

Year	Primary Production	Secondary Production	Consumption	Real Prices Aluminum	Real Prices Copper
1947	1049.8	.	1093.8	18.3007	30.7058
1948	1224.0	442.7	1246.1	17.7935	29.1160
1949	1255.6	335.0	1208.7	20.3304	21.1308
1950	1505.9	429.8	1581.6	20.4315	27.3233
1951	1807.6	475.1	1807.6	19.7585	30.0462
1952	2032.3	483.3	1954.8	20.7788	36.7206
1953	2453.9	558.3	2386.8	22.5458	33.4783
1954	2820.4	523.0	2540.3	23.0639	35.2822
1955	3104.7	611.1	3104.4	24.9260	50.0990
1956	3337.6	611.2	3221.3	26.4961	45.0982
1957	3387.2	648.8	2978.3	27.2412	30.0305
1958	3526.7	618.6	3115.9	26.2051	26.1046
1959	4021.6	727.6	3962.1	26.0949	31.3503
1960	4476.2	753.9	4087.4	27.3973	32.4411
1961	4517.1	931.9	4426.9	26.9397	30.4543
1962	4922.2	1061.9	4880.5	25.1846	30.8801
1963	5319.9	1205.1	5357.8	23.9397	30.9537
1964	5977.0	1327.9	5899.2	25.0697	46.1738
1965	6500.6	1510.1	6520.3	25.3696	60.6216
1966	7118.7	1657.4	7518.7	24.5491	69.2653
1967	7843.1	1699.6	7646.7	24.9780	44.9188
1968	8538.8	1946.3	8704.3	24.9590	54.5526
1969	9329.3	2255.8	9467.1	25.5174	62.4649
1970	10296.6	2180.6	9796.2	26.0109	57.0581
1971	10791.0	2295.2	10569.1	25.4609	44.4559
1972	11494.0	2447.8	11524.6	22.1738	37.6546
1973	12717.3	2808.3	13412.9	18.5598	55.9842
1974	13647.5	2881.6	13549.9	21.3198	57.4728
1975	12537.5	2588.1	10979.3	22.7479	28.7455
1976	12897.4	3078.8	13605.8	24.2301	32.4167
1977	14000.7	3244.8	13852.6	.	.

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[1] See Fisher, Cootner, and Baily (1972).

[2] See Banks (1974) and Fisher, Cootner, and Baily (1972).

FIGURE 7-1
ALUMINUM PRIMARY PRODUCTION
NORTH AMERICA (▲), EUROPE (■)
CENTRALLY PLANNED (●)
(thousands of metric tons)

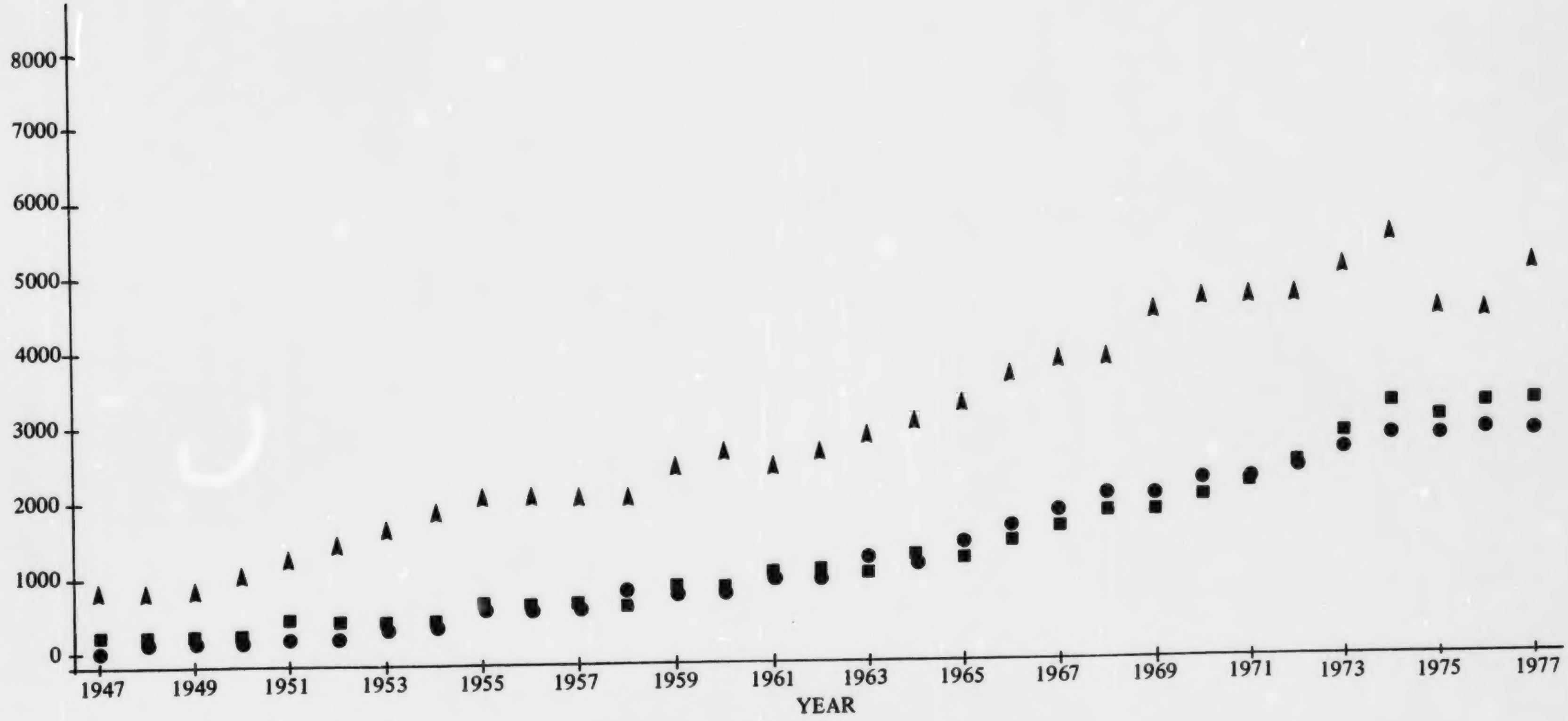


FIGURE 7-2
ALUMINUM PRIMARY PRODUCTION
SOUTH AMERICA (▲), AFRICA (■), OCEANIA (●)
(thousands of metric tons)

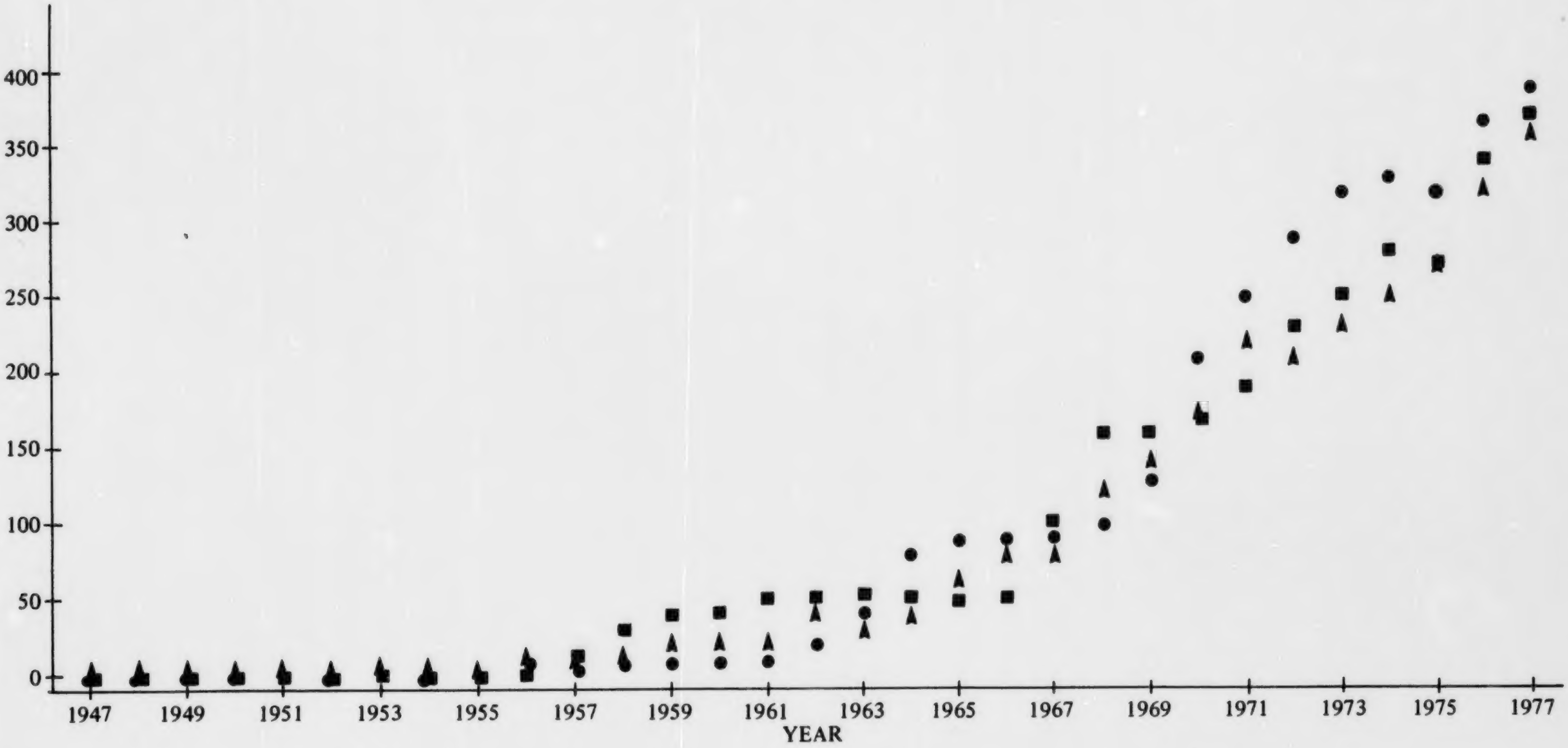


FIGURE 7-3
ALUMINUM PRIMARY PRODUCTION
ASIA (▲)
(thousands of metric tons)

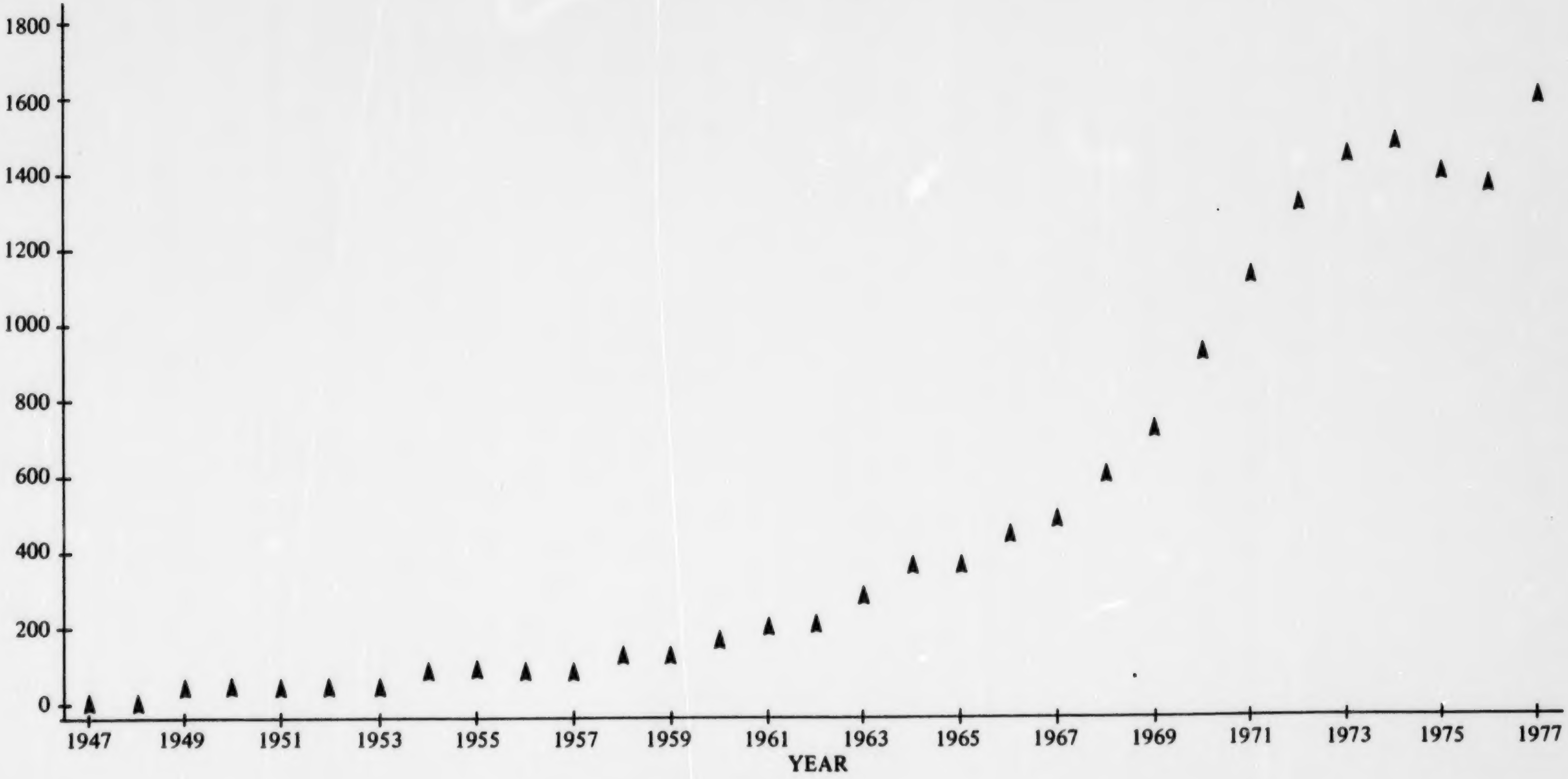


FIGURE 7-4
ALUMINUM CONSUMPTION
NORTH AMERICA (▲), EUROPE (■)
CENTRALLY PLANNED (●)
(thousands of metric tons)

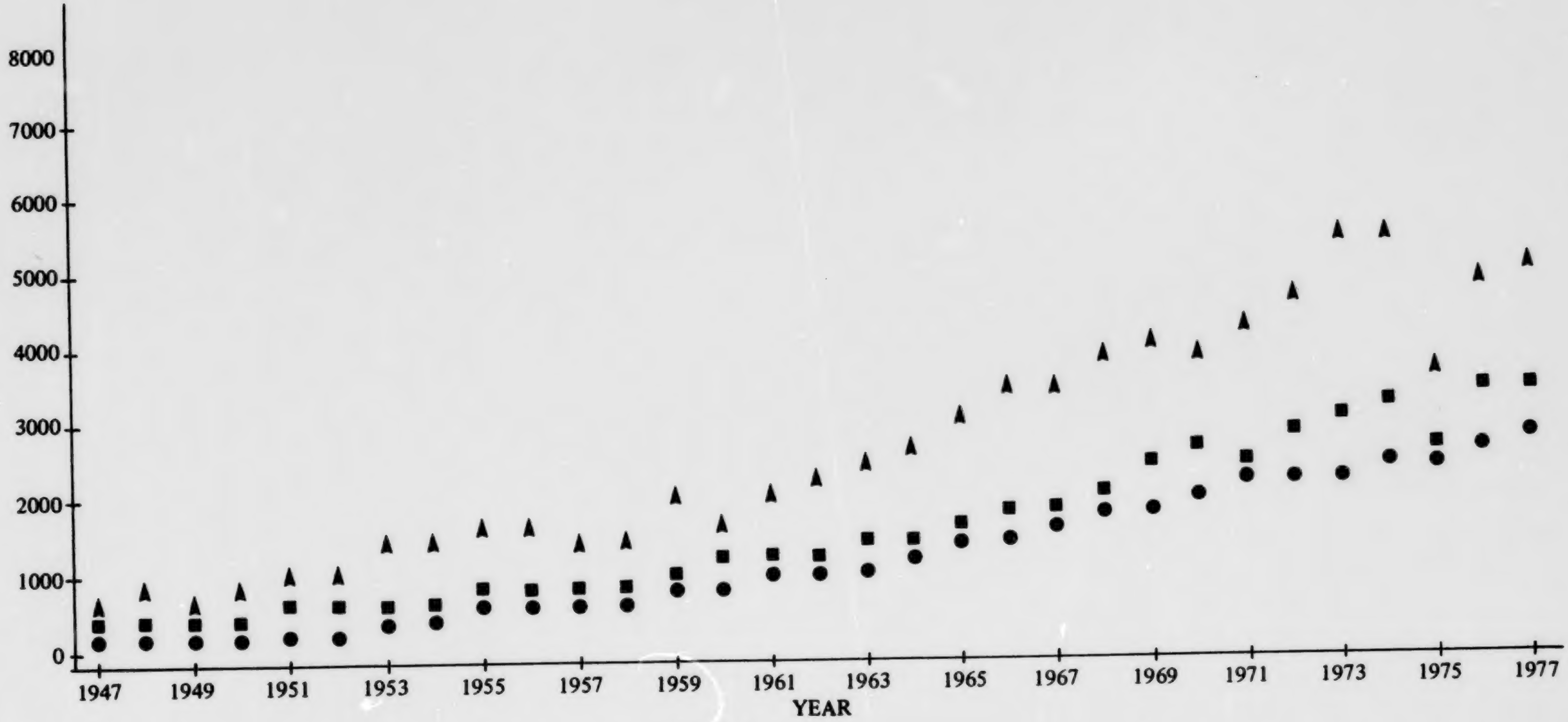


FIGURE 7-5
ALUMINUM CONSUMPTION
SOUTH AMERICA (▲), AFRICA (■), OCEANIA (●)
(thousands of metric tons)

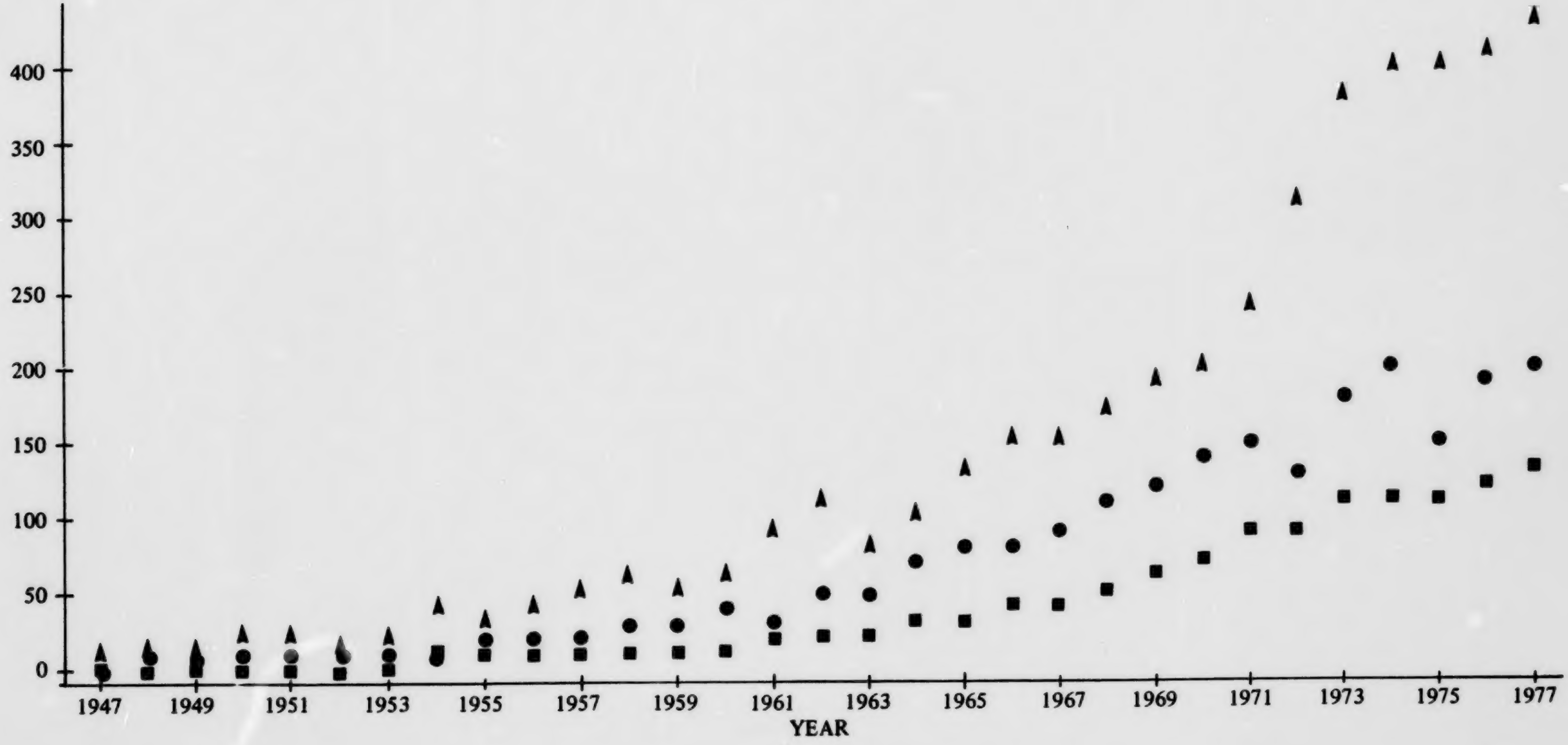
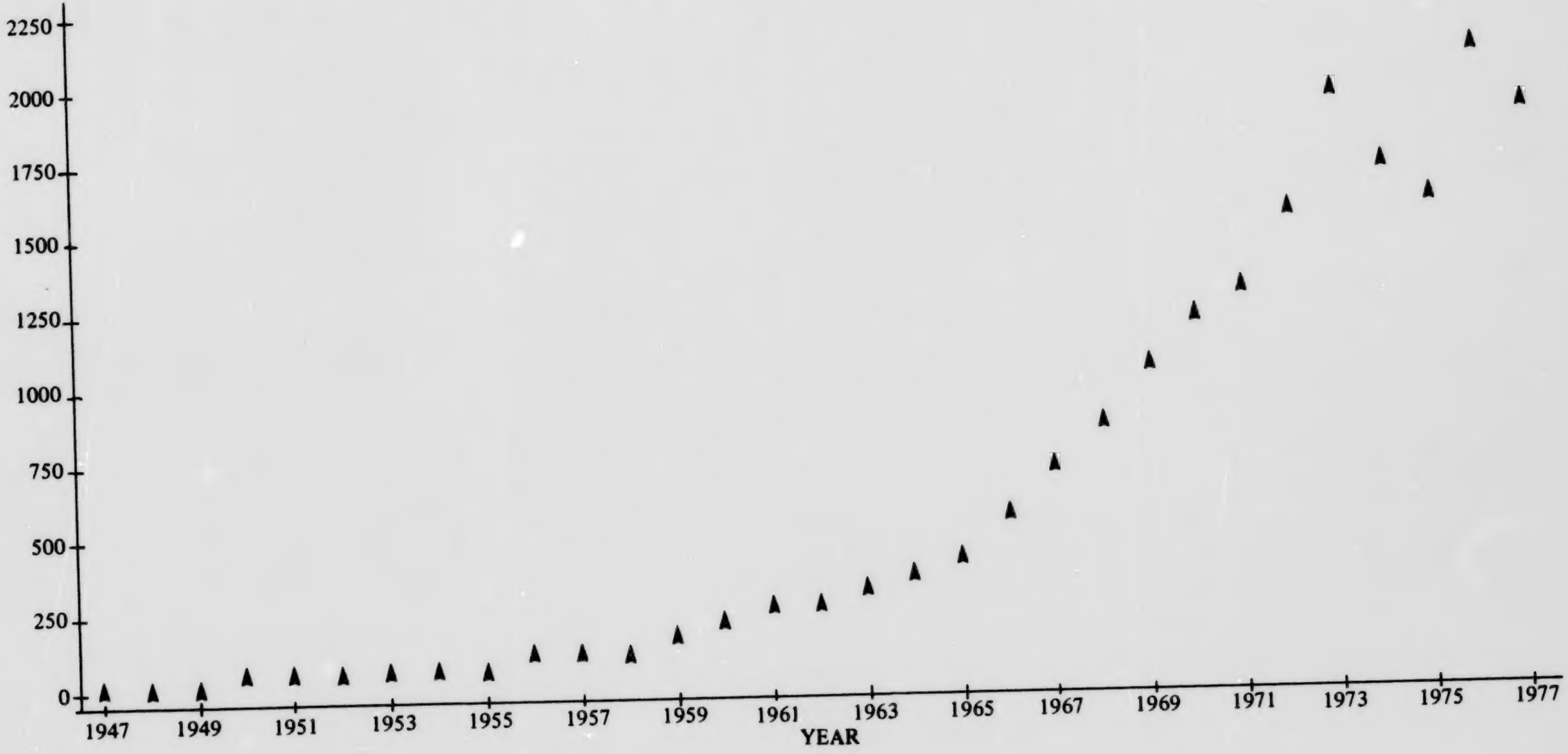


FIGURE 7-6
ALUMINUM CONSUMPTION
ASIA (▲)
(thousands of metric tons)



7.3 EMPIRICAL RESULTS

The econometric equations for aluminum were estimated using the data described in Chapter 5 and in the preceding section of this chapter. These results are presented in Tables 7-5 through 7-8. Following is a brief discussion of these estimates.

TABLE 7-5
ESTIMATED EQUATIONS
PRIMARY PRODUCTION
ALUMINUM
(T-STATISTICS IN PARENTHESES)

		North America 1952-1976	South America 1952-1976	Asia 1952-1976	Africa 1959-1976	Europe 1952-1976	Oceania 1952-1976	Centrally Planned 1952-1976
θ_0	Constant	-2897.06	137.258	537.813	198.285	840.791	-5.5657	-2840.19
		(-1.0042)	(1.0496)	(1.0576)	(0.4348)	(1.4506)	(-0.0085)	(-3.2146)
θ_1	$P_{M,t}$	30.6477	4.9239	1.8037	1.9218	1.6431	4.3082	15.1127
		(1.2525)	(2.1210)	(0.1842)	(0.4805)	(0.1462)	(0.5224)	(0.9219)
θ_2	$P_{M,t-1}$	-2.8914	-6.1849	-0.3574	-0.9512	-7.8384	1.3817	-1.6014
		(-0.1291)	(-2.3179)	(-0.0380)	(-0.1432)	(-0.5991)	(0.1451)	(-0.0838)
θ_3	$ROWS_{M,t-1}$	-0.4123	0.0552	0.1274	-0.0738	0.0355	0.0064	0.1212
		(-1.3622)	(3.2533)	(2.4524)	(-1.9831)	(0.4005)	(0.1148)	(0.7933)
θ_4	$D_{M,t}$	0.3784	0.0055	0.0324	0.0129	0.0579	0.0187	0.0868
		(9.2631)	(1.0814)	(2.1918)	(1.5221)	(1.7803)	(1.3709)	(2.4060)
θ_5	$D_{M,t-1}$	0.3548	-0.0356	-0.0738	-0.0198	0.0396	-0.0049	-0.0022
		(2.6701)	(-3.3244)	(-1.9831)	(-0.6908)	(0.5581)	(-0.1371)	(-0.0233)
θ_6	t	47.9072	-3.3201	-14.4830	-4.9218	-13.0657	-4.0795	46.8268
		(0.7889)	(-1.0494)	(-1.3298)	(-0.5659)	(-1.0273)	-0.4025)	(2.2712)
θ_7	r_t	-10.8994 (-0.1439)	NA	NA	NA		-5.7795 (-0.2336)	NA
θ_8	r_{t-1}	161.381 (1.9133)	NA	NA	NA		10.6340 (0.3077)	NA
θ_9	$\sum^{t-2} Q^S_{M,i}$	-0.0429	-0.0412	-0.0837	0.0211	0.0280	-0.0101	0.0013
		(-1.0168)	(-1.3445)	(-3.3931)	(0.2907)	(0.6075)	(-0.0841)	(0.0434)
θ_{10}	$Q^S_{M,t-1}$	-0.1359	0.6086	1.2366	0.4960	0.3341	0.7864	-0.1936
		(-0.6435)	(2.3569)	(5.0120)	(1.5652)	(0.7161)	(1.8691)	(-0.6598)
	R^2	0.9965	0.9931	0.9972	0.9794	0.9979	0.9839	0.9946
	SEE	94.9076	10.3483	33.3499	19.6851	50.5061	23.7600	76.7957

7.3.a PRIMARY PRODUCTION

All of the parameters in the primary supply function for North America carried the hypothesized signs. It is, however, interesting to note the fact that the price parameters were statistically insignificant while those for demand were very significant. Indeed, these demand variables appear to be the primary determinants of primary production. While not conclusive, these results are consistent with the assertion that the U.S. aluminum price is to some degree administered.[3] Examination of the rest of Table 7-5 indicates that only in South America were the price terms significant.

The primary supply function for South America proved very satisfactory. Not only were all of the parameters of the hypothesized sign, but also the parameters were found to be statistically significant.

In the case of Asia, all of the signs were again as hypothesized. Although the demand variables were shown to have significant effect, it would appear that there exists a strong trend in primary production from this country group as indicated by the estimate for $Q^S_{M,t-1}$.

The function for Africa carried all of the hypothesized signs except for previous production which was statistically insignificant. It would appear from these estimates that African production has a strong trend component modified by price and consumption expectations.

In our estimation of primary supply from Europe, the interest rate terms were excluded because estimates including these variables proved to be unsatisfactory. Also, the parameter for previous extraction($\sum^{t-2} Q^S_{M,i}$)had a sign opposite of that predicted but it was statistically insignificant. Examination of the other parameters indicated that demand (consumption) has the most significant effect on primary production.

For Oceania, all of the parameters carried the hypothesized signs. As with Europe, demand was found to be statistically significant; there is also evidence of a significant trend.

Finally, in the case of the Centrally Planned Economies, only the parameter for previous production (θ_9) had a sign opposite of that predicted; the parameter was statistically insignificant. For this country group, primary production is apparently strongly influenced by trend (t), modified by current consumption.

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[3] See Fisher, Cootner, and Baily (1972).

TABLE 7-6
ESTIMATED EQUATIONS
SECONDARY PRODUCTION
ALUMINUM
(T-STATISTICS IN PARENTHESES)

		North America 1952-1976	South America 1952-1976	Asia 1952-1976	Africa 1952-1976	Europe 1952-1976	Oceania 1952-1976
ϕ_0	Constant	-54.7408	2.9039	-10.7487	-0.9371	-29.7566	-1.7844
		(-0.3160)	(0.3404)	(-0.5964)	(-0.5026)	(-0.6897)	(-0.4809)
ϕ_1^P	$P_{M,t}$	2.8214	0.2940		0.0207		
		(0.2653)	(0.4548)		(0.2020)		
ϕ_2^P	$P_{M,t-1}$	-1.0774	-0.5360		0.0335	1.8970	
		(-0.1215)	(-0.9812)		(0.3668)	(1.1189)	
ϕ_3^P	$P_{M,t-2}$			0.4498			0.0245
				(0.6007)			(0.1523)
ϕ_1^D	$Q_{M,t}^D$	0.1050	0.1267	0.2167	0.0142	0.2579	0.1220
		(3.6202)	(3.1716)	(14.1865)	(0.5576)	(11.8466)	(3.7002)
ϕ_2^D	$Q_{M,t-1}^D$	-0.0716	0.0176	0.0047	0.0014	-0.1538	-0.0357
		(-2.1456)	(0.3202)	(0.0965)	(0.0556)	(-2.4600)	(-0.8335)
ϕ_3^D	$Q_{M,t-2}^D$	0.0569	-0.0632	-0.1365	-0.0050	0.0164	-0.0795
		(1.7861)	(-1.5786)	(-4.1599)	(-0.2114)	(0.7328)	(-2.6943)
ϕ_4^D	$Q_{M,t-3}^D$			0.0213			0.1173
				(1.1178)			(2.4312)
ϕ_S	$Q_{M,t-1}^{SCRAP}$	0.6935	0.3777	0.6317	0.8835	0.6279	0.3942
		(3.8695)	(1.8161)	(3.2424)	(5.9443)	(2.9754)	(1.8799)
	R^2	0.9801	0.9841	0.9990	0.9744	0.9966	0.9851
	SEE	59.1218	2.7135	7.0244	0.6042	18.5207	1.5857

7.3.5 SECONDARY PRODUCTION

For aluminum scrap, data were available for all of the country groups except the Centrally Planned Economies. The estimates for the secondary supply functions are presented in Table 7-6.

In the case of North America, all of the parameters had the hypothesized signs. Examination of the parameters indicated that both new and old scrap were significant in scrap production. The parameters for both current and lagged consumption were statistically significant. As with all of the country groups, North American scrap production showed a significant trend.

For South America, all of the parameters again carried the hypothesized signs. While not conclusive, these estimates tend to indicate that scrap production is more strongly influenced by new scrap production.

As shown in Table 7-6, current and once lagged price proved unsatisfactory in the secondary supply function for Asia. Indeed, the price variable included was not indicated to be significant. Instead, scrap production in Asia appears to be only a function of current consumption (new scrap availability) and lagged consumption (old scrap availability).

In the case of Africa, the sign for the parameter of once lagged price was opposite that predicted; it was not statistically significant. It would appear from the parameter estimates that African scrap production is determined solely by trend.

In our estimation for Europe, only once lagged price was included. Although it had a sign other than predicted, the fact that current price was excluded would tend to make this parameter indeterminant. It would appear from the parameter estimates that in Europe, as in North America and Asia, old scrap is a significant component of the secondary supply.

Estimations for Oceania indicated that only twice lagged price should be included. Since only the demand variables and trend had significant parameters, it would again appear that secondary production is determined by new and old scrap availability.

An overall evaluation of Table 7-6 indicates that aluminum scrap production is determined by availability. That is, the price variables are all statistically insignificant and scrap production is determined by current and lagged consumption (availability) as well as lagged scrap production (trend).

TABLE 7-7
ESTIMATED EQUATIONS
DEMAND
ALUMINUM
(T-STATISTICS IN PARENTHESES)

		North America 1952-1976	South America 1962-1976	Asia 1962-1976	Africa 1962-1976	Europe 1952-1976	Oceania 1952-1976	Centrally Planned 1952-1976
Ψ_0	Constant	-7442.93	-576.526	-1311.98	-277.958	-775.586	-314.555	-176.866
		(-4.7027)	(-2.4323)	(-1.5984)	(-2.4312)	(-1.4194)	(-6.1073)	(-1.7498)
Ψ_1	$P_{M,t}-P_{R,t}$	-14.6924	-0.5681		-0.1813	-6.3905		-1.2201
		(-2.2299)	(-1.1073)		(-0.9958)	(-1.8569)		(-1.1343)
Ψ_2	$P_{M,t-1}-P_{R,t-1}$	17.4787	1.3230		0.0787	3.9309	-0.0890	-1.7754
		(2.7127)	(2.1113)		(0.4532)	(0.9198)	(-0.3646)	(-1.4940)
Ψ_3	$S_{M,t-1}-D_{M,t-1}$	-0.5254	-0.0318		0.0048	0.0516	0.0006	0.0421
		(-1.3475)	(-1.5144)		(0.7481)	(0.5393)	(0.0888)	(1.1909)
Ψ_4	Y_t	9832.74	12.5735	6395.91	145.537	6105.14	221.562	1193.00
		(2.8188)	(0.0264)	(3.7401)	(1.5643)	(2.7929)	(1.9654)	(1.4652)
Ψ_5	Y_{t-1}	3504.77	823.674	-4528.01	205.501	-4466.15	263.590	-260.201
		(0.9192)	(1.1014)	(-2.7539)	(1.4565)	(-1.5497)	(1.6138)	(-0.2801)
Ψ_6	$\dot{K}_{t-1}$	1486.39	7.7562	1471.78	-33.0556	-862.369	11.3677	NA
		(0.9017)	(0.0472)	(1.3197)	(-0.3868)	(-0.9706)	(0.1963)	
Ψ_7	$\dot{M}_{t-1}$	-73.7263	NA	NA	NA	10.5161	0.9156	NA
		(-1.5090)				(0.7739)	(2.2923)	
Ψ_8	$Q^D_{M,t-1}$	-0.3829	0.1024	0.1720	0.1267	0.5822	-0.3256	0.5942
		(-1.3341)	(0.2767)	(0.3935)	(0.3658)	(1.9999)	(-1.5894)	(2.9924)
	R^2	0.9794	0.9888	0.9769	0.9852	0.9863	0.9835	0.9967
	SEE	238.704	17.9878	111.624	6.2653	127.562	9.4081	49.4283

7.3.c DEMAND

In the case of North America, only the sign of the parameter for lagged consumption was opposite to that predicted theoretically. Both the price and output terms are indicated to be statistically significant determinants of quantity demanded.

The estimated demand function for South America carried all of the predicted signs. However, an examination of these parameters indicated that only the lagged prices and the inventory effect $(S_{M,t-1}-D_{M,t-1})$ were significant. It appears that South American consumption has the strong flavor of planned consumption (i.e., only the variables that determine expected price are significant).

Our estimations indicated the price variables to be unsatisfactory in the case of consumption in Asia. Therefore they were excluded. Our results indicate that the demand for aluminum in Asia is determined solely by the demand for final commodities, reflected in actual and expected per capita income. This result is not surprising given the rapid growth in Asia (especially Japan) over the sample period.

For Africa, all of the estimated parameters carried the hypothesized signs. However, examination of the individual parameters indicated that only the income variables were significant.

The estimated demand function for Europe proved very satisfactory. All of the parameters were as hypothesized and most were statistically significant. In this country group the estimated parameters for current prices and current income indicated a significant adjustment to unanticipated market conditions.

For Oceania, current prices proved unsatisfactory and were excluded. Furthermore, the sign on lagged consumption was opposite that which we had predicted. The remaining coefficients indicated that, for this country group, aluminum consumption is primarily determined by the demand for final goods that use aluminum as an input. The significance of the parameter for current income indicates Oceania's ability to adjust to unanticipated demand conditions.

In the case of the Centrally Planned Economies, all of the estimated parameters carried the predicted signs. While price and income apparently exert some effect, it should be noted from the parameter for lagged consumption that there exists a significant trend in the demand for aluminum.

Overall, these results indicate that the consumption of aluminum is primarily determined by the demand for final commodities that use aluminum as an input. Only in North America and Europe was there significant evidence that users' plans were altered in response to unanticipated prices.

TABLE 7-8
ESTIMATED EQUATIONS
PRICE
ALUMINUM
(T-STATISTICS IN PARENTHESES)
1952-1976

η_0	Constant	-3.8888 (-0.3178)
η_1	$P_{M,t-1}$	0.3738 (1.0976)
η_2	$P_{M,t-2}$	-0.2233 (-0.8600)
η_3	$S_{M,t-1}-D_{M,t-1}$	0.0018 (1.1698)
η_4	t	0.5961 (1.9655)
η_5	r_t	1.6056 (2.4030)
η_6	$\sum^{t-2} S_{M,i}$	-0.00003 (-0.4263)
η_7	$WQ^{SCRAP}_{M,t-1}$	-0.0049 (-1.5518)
η_8	$P_{R,t}$	0.0223 (0.6896)
η_9	$P_{R,t-1}$	0.0855 (2.2187)
η_{10}	$P_{R,t-2}$	
η_{11}	Y_t	-19.7233 (-1.5973)
	R^2	0.8747
	SEE	0.9999

7.3.d MARKET PRICE

Our estimated price equation for aluminum is displayed in Table 7-8. Again, no hypotheses were made concerning the signs of the parameter in this equation. However, it can be seen that there exists evidence to support the idea of a trend in the price of aluminum modified by the price of copper and by income.

- 8 -
NICKEL

8.1 AN OVERVIEW OF THE NICKEL MARKET

The Ministry's current program of mineral markets analysis and simulation started with certain suspicions regarding the conventional interpretation of nickel market behaviour which were raised in the minds of Ontario Ministry of Natural Resources staff in the course of an industry presentation of their nickel market analysis in May of 1974. Their then conventional approach to world nickel consumption data series from 1945 consisted of a simple log-linear regression analysis which yielded an approximately 6% per year compound growth rate in consumption. Extrapolated into the future this implied the need for nickel production capacity in the year 2000 in the plus 2.5 million metric ton range. Looking at the same data points, we perceived a different pattern: first fairly slow growth from the end of World War II until the late 1950s, then very steep growth from the late 1950s to the mid 1960s and then a slowing down. If the flat portions during the first and third period were characteristic of long growth and if the steep portion in the middle period was unusual, a quite different path of consumption growth to the year 2000 would be indicated, a path which would imply requirements of a year 2000 world production capacity not significantly exceeding the capacity estimated then to be in place by 1980, e.g. in the 1.1 to 1.2 million metric ton range.

Our interpretations were immediately conveyed to and discussed with various industry analysts who were, however, not convinced. It was decided to carry out further work on the behaviour of some non-ferrous metal markets. In December 1977 the Ministry of Natural Resources issued a commodity study (Ontario Ministry of Natural Resources Mineral Policy Background Paper No. 4) entitled: *Toward a Nickel Policy for the Province of Ontario.* In this study 3 causes were considered to be potentially responsible for the steep consumption growth during the middle period. These were high military consumption in the sequence Korea, Cold War, Vietnam; catch-up in per capita consumption to levels of the U.S.A. among the Western European nations and Japan, and lastly the probability that there had been inflationary overinvestment in the capital goods sectors. The first one of these causes was, after some consideration, dismissed as being most likely insignificant. To test the implications of the second factor, some calculations were carried out which, as it turned out, were based on a concept similar to the intensity-of-use concept used by Professor Malenbaum in 1978, in *World Demand For Raw Material In 1985 and 2000*, and which supported our hypothesis. The third factor was considered important in light of Austrian cyclical theory and particularly of the account given by Professor Bresciani-Turroni in *The Economics of Inflation* of the effects of the end of the great German inflation of the 1920s on the German mining and capital goods sectors.

Let us now look at actual data of nickel consumption since 1977 and compare them with the "most probable projection" given in our 1977 publication, which was based on data series ending with the year 1976. Actual 1977 consumption was 5.5% below the line connecting actual 1976 consumption with the mid-point of the year 2000 "most probable projection", actual consumption in 1978 was exactly on line, actual consumption 1979 was 8.6% above this trend and actual 1980 consumption 4.9% below this trend.

By the time the nickel commodity study was published it had already become apparent that the liquidation of excess inventories, which in the case of our major nickel producers alone approached the $1 billion range, had become a major corporate and government policy problem. In fact it was not until late 1980 that Inco's inventories finally were reduced to what is considered a normal range. This inventory problem affected labour issues, views on the ability of industry to absorb higher environmental expenditures, on regional development, and on taxation on the government side as well as forward planning on the corporate side.

While the results of erroneous long term forecasting already provided a painful lesson to corporate and government policy makers, the myth of the 6% and even higher compound growth rate still prevailed in the corridors of the United Nations, particularly with reference to The Law of the Sea Conference. At the time the nickel commodity study was published it had become obvious that a more sophisticated analysis of the same issue had to be carried out and this was published in May 1979 as Mineral Policy Background Paper No. 8, "*World Mineral Markets: An Econometric and Simulation Analysis*", the first stage of the current program.

In 1979 a number of forecasters, particularly the Japanese following the lead of Sumitomo, were beginning to reduce their forecast growth rates. In 1980, Inco officially lowered its forecast consumption growth rate from 6% to 4% and in 1981 discussed 4% as an optimistic upper limit of potential consumption growth. As we shall see later the time has now come to lower forecasts from a 1980 starting base to a rate possibly as low as .5% or perhaps a slightly better 1%, through the 1980's if not the rest of this century.

The most important use of nickel is as an alloy in steel fabrication. These uses have been discussed in the nickel commodity study already referred to. Nickel and steel being largely complements, nickel consumption may be expected to reflect levels of capital investment in the industrialized countries of the world.

Nickel prices through the 1970's have in real terms been higher than at any time since 1902 with the exception of the first 3 years of the first World War. During the 1970s there was a small dip in 1974 and a much more severe dip in 1978, but even in 1978 the real prices only sank to the level of the early 1930's. The major moves of nickel prices were a long period of decline from 1914 to 1920, a sharp peak in 1921 with a subsequent decline by 1923 to the 1920 level, then a prolonged period of increases until 1932, a 16-year long decline from 1932 to 1948, a gradual recovery from 1948 to 1957, a period of very stable prices from 1957 to 1966 and then the increase from 1966 to the 1970's plateau. As far as the 1980's and possibly the 1990's are concerned, a significant reduction in real prices from the level of the 1970's is a distinct possibility, which would reflect increasing competitiveness in the nickel market and increasing supplies from the Centrally Planned Economies, and which would involve reductions in world-wide capacity of between 20% and 30% from current installed capacity levels.

If this rather grim outlook for real nickel prices is disturbing, a look at the growth scenarios provided in the 1979 analysis as well as those provided subsequently in Chapter 10 may provide some light.

The simulations in the first phase study for all scenarios, except those diverging from trend by two standard deviations, showed consumption and production to be reasonably close. In the simulations of the second phase, production under all scenarios was significantly higher than consumption. This reflects primarily the changes in the model which provided for greater realism in the treatment of inventory accumulation. The scenarios in this second phase clearly indicate the danger of a recurrence of the problem of excessive inventory accumulation of the early 1970's.

The addition of new data highlights particularly the declining influence of the 1973/1974 period of apparent resumption of rapid growth in the consumption data series. It shows up very well if linear trend extrapolations to the year 2000 are compared for the series 1970/1980; 1973/1980; 1973/1981 (est) and 1973/1982 (est). Year 2000 consumption levels for these four series are respectively 1.092; .971; .920 and .881 million metric tons (total world).

In the first phase study consumption growth scenarios higher than the base simulation were very clearly accompanied by price trends lower than the base simulation and consumption growth rates lower than the base simulation by price trends higher than the base simulations.

In this second phase price trends are above the base simulation for growth scenarios above the base simulation and price trends are lower than the base simulation for growth scenarios below the base simulation. In the earlier study the price trends for low and high growth scenarios diverged increasingly from the base trend. In this second study, high and low growth scenario price trends run almost parallel and very close to the base trend. Both approaches reflect the fact that the market had in the past been very clearly dominated by Inco. The differences are accounted for by changes in the structure of the model.

In both phases of the study a changed base scenario was provided under which it was assumed that the nickel market would become quite highly competitive; that is, that it would begin to approach the characteristics of the copper market. The base scenario under these changed assumptions showed in the first study a real price rise for nickel of only 5.93% from 1980 to 1988 compared with the real price rise under the price leadership base simulation of 19.3% for the same period. For the scenario providing a decline in growth rates of income and capital acquisition in Asia, Europe and the Centrally Planned Economies relative to those for North America, a price rise of 2.39% was generated under competitive conditions compared with 22.38% under price leadership conditions.

In the second stage, prices, under the base scenario, rose from 1980 to 1991 by 39% in real terms and under the modified scenario declined over the same period by .1% after rising to a peak in 1986 which exceeded the 1980 level by 7.51%. Considering that in the current phase the structure of the nickel market has only changed to a limited extent and that the base scenario still allows for a massive excess of production over consumption, a still more realistic scenario would have to apply the highly modified market structure to a low growth scenario and impose, as an external shock to the system, significant cutbacks in production. How this might be accomplished with the model will be the subject of the subsequent third stage report on this project.

The model cannot of course predict how and where such cuts are likely to be made, particularly as increasing amounts of nickel are being produced in non-market economies. Further, "most likely" scenarios must be chosen on the basis of considerations not part of the model and interpreted in light of the results of alternative approaches and of experts' knowledge of the details of the industry.

We do feel sure, however, that consumption growth over this decade will be very slow, that there will be at best insignificant increases in real prices, and that considerable capacity reductions will be implemented.

8.2 DATA

In addition to the country group data described in Chapter 5, data were required for primary production, secondary production, and consumption, as well as the world price of nickel and of the related commodity. In this analysis, we will use, as the related commodity, raw steel.[1] The data for the price of raw steel were obtained from the *American Metal Market* statistics and the remainder of the data were collected from *Metal Statistics*. Following is a brief description of these data.

For primary production, we employed the data reported on mine production, expressed in thousand metric tons. The country specific data are presented in Table 8-1 and plotted in Figures 8-1 and 8-2. Aggregating over the country groups, world primary production is reported in column 1 of Table 8-4.

In the case of nickel, secondary (scrap) production (i.e., scrap recovery) was reported only for the United States. We will, in our analysis, treat this as world secondary production. These data are presented in Table 8-2 and repeated in column 2 of Table 8-4.

Data for the consumption of refined nickel in thousand metric tons were also collected. These data, by country group are displayed in Table 8-3 and plotted in Figures 8-3 through 8-5. World consumption of nickel (i.e., the sum of the individual country group data) is reported in column 3 of Table 8-4.

As the world price of nickel, we used the U.S. price for electrolytic cathodes denominated in cents per pound. In order to convert these nominal prices into real prices, we deflated these values by the U.S. wholesale price index (1967=100). These data are presented in column 4 of Table 8-4.

The world price of raw steel was defined to be the U.S. steel billet price in dollars per net ton. These values were also deflated by the U.S. wholesale price index (1967=100). The resulting real prices for steel are shown in column 5 of Table 8-4.

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[1] While it is clear that certain types of steel and nickel are used together in production, it is not clear that such a relation always holds. Thus we will not hypothesize *a priori* about the substitutability or complementarity of steel and nickel.

Table 8-1
Nickel: Primary Production
(thousands of metric tons)

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1947	107.6	2.6	0.0	0.6	0.5	2.9	16.2
1948	119.5	0.8	0.0	0.6	0.2	3.6	17.8
1949	116.7	0.7	0.0	0.6	0.1	2.5	25.4
1950	112.2	0.8	0.0	0.9	0.4	4.3	29.6
1951	125.1	0.7	0.6	1.2	0.1	6.7	33.7
1952	127.5	8.7	0.1	1.5	0.4	10.7	37.8
1953	131.0	12.7	0.1	1.8	0.2	17.1	41.0
1954	146.7	13.2	0.1	2.0	0.1	13.7	43.0
1955	162.2	13.9	0.2	2.0	0.1	24.7	45.1
1956	168.0	14.7	0.1	3.5	0.8	29.5	48.5
1957	179.6	20.3	0.1	3.4	1.1	43.3	51.6
1958	137.3	18.0	0.4	1.4	0.4	14.2	54.9
1959	179.7	17.9	0.1	2.8	0.3	32.8	55.0
1960	206.0	14.6	0.2	3.2	2.1	53.0	60.3
1961	221.5	16.6	0.5	2.9	2.0	54.0	62.8
1962	225.2	20.1	0.5	3.8	2.4	34.5	65.0
1963	207.3	1.0	1.1	3.6	2.9	44.5	102.7
1964	218.4	1.1	1.2	4.6	3.2	58.2	105.3
1965	247.4	1.1	2.4	6.3	3.0	61.2	110.2
1966	214.9	1.1	2.6	6.5	3.0	67.8	119.7
1967	338.9	1.1	3.9	6.9	5.9	84.8	134.5
1968	253.5	1.1	5.7	9.0	7.4	120.8	147.6
1969	208.1	1.6	5.0	13.0	9.2	128.0	147.5
1970	291.6	2.9	10.9	22.6	13.6	168.3	155.5
1971	281.2	3.0	15.0	24.5	14.6	186.4	155.8
1972	249.2	20.8	14.5	23.9	17.0	143.6	156.2
1973	265.6	68.0	16.3	32.0	20.1	156.0	124.5
1974	284.2	68.0	16.4	40.2	21.5	182.7	130.0
1975	257.6	67.4	24.2	32.7	20.9	209.1	136.7
1976	255.7	66.5	32.7	51.2	23.2	201.4	142.5
1977	248.4	67.4	53.0	47.0	15.8	201.2	148.0

Table 8-2
Nickel: Secondary Production

Year	Scrap
1947	8.655
1948	8.029
1949	5.153
1950	7.979
1951	7.804
1952	6.785
1953	7.578
1954	7.806
1955	10.470
1956	13.481
1957	10.920
1958	6.723
1959	8.562
1960	8.556
1961	9.691
1962	10.077
1963	17.233
1964	20.969
1965	17.606
1966	24.292
1967	18.807
1968	12.756
1969	17.032
1970	21.010
1971	24.345
1972	32.592
1973	29.601
1974	18.987
1975	16.220
1976	12.041
1977	.

Table 8-3
Nickel: Consumption
(thousands of metric tons)

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1947	74.8	0.3	0.5	0.0	28.7	0.8	17.0
1948	86.6	0.4	0.5	0.0	28.6	0.3	18.5
1949	63.8	0.2	0.8	0.0	28.0	1.2	25.5
1950	93.6	0.2	0.8	0.0	32.0	0.5	30.0
1951	81.9	0.3	1.0	0.0	36.9	0.3	34.0
1952	95.9	0.3	1.3	0.0	38.0	0.6	38.0
1953	100.0	0.3	2.0	0.0	38.4	0.6	41.0
1954	89.7	0.5	2.5	0.0	45.0	0.5	43.0
1955	104.9	0.5	3.5	0.1	52.2	0.6	45.1
1956	120.9	0.9	5.9	0.2	53.8	0.7	48.5
1957	115.5	0.8	6.7	0.3	59.5	0.9	51.6
1958	75.6	0.5	7.1	0.2	55.3	0.7	56.0
1959	105.9	0.5	11.0	0.3	69.8	1.3	59.0
1960	103.2	0.7	15.4	0.3	94.7	2.2	74.0
1961	114.7	1.2	21.3	0.5	88.9	1.2	80.0
1962	115.7	1.2	16.7	0.5	80.0	1.2	85.0
1963	118.2	1.5	24.5	0.5	87.7	2.0	108.4
1964	139.6	1.5	31.8	1.0	112.9	2.6	108.4
1965	164.2	1.5	27.7	2.0	120.2	2.5	110.5
1966	178.2	1.4	37.6	3.0	129.0	2.9	115.5
1967	165.7	1.5	51.7	4.0	131.1	3.5	115.5
1968	154.7	2.0	61.0	4.0	144.9	3.0	120.8
1969	149.8	1.8	76.4	4.0	147.0	2.9	120.9
1970	164.1	3.1	101.5	3.6	174.2	4.1	126.0
1971	144.5	2.9	94.2	3.5	146.8	3.9	130.8
1972	169.4	3.7	90.7	3.2	163.2	3.6	125.9
1973	184.7	12.8	115.7	5.0	181.1	5.0	126.6
1974	200.7	10.8	123.4	6.0	202.2	5.0	137.2
1975	144.2	7.7	89.0	6.0	151.8	2.0	150.3
1976	159.3	9.3	127.6	5.0	179.1	4.0	159.6
1977	148.9	11.8	106.2	5.2	182.8	4.3	165.5

Table 8-4
Nickel
World Primary and Secondary Productions
World Consumption and Prices

Year	Primary Production	Secondary Production	Consumption	Real Prices	
				Nickel	Steel
1947	130.4	8.655	122.1	45.752	56.536
1948	142.5	8.029	134.9	44.123	58.128
1949	146.0	5.153	119.5	50.826	66.112
1950	148.2	7.979	157.1	54.758	65.098
1951	168.1	7.804	154.4	59.276	61.471
1952	186.7	5.785	174.1	63.770	64.673
1953	203.9	6.785	182.3	68.522	69.348
1954	218.8	7.806	181.2	69.010	71.895
1955	248.2	10.470	206.9	74.601	75.399
1956	265.1	13.481	230.9	71.847	77.960
1957	299.4	10.920	235.3	79.314	81.190
1958	226.6	6.723	195.4	78.224	83.002
1959	288.6	8.562	247.8	78.059	84.388
1960	339.4	8.556	290.5	77.977	84.299
1961	360.3	9.691	307.8	82.172	84.656
1962	351.5	10.077	300.3	84.277	84.388
1963	363.1	17.233	342.8	83.598	85.735
1964	392.0	20.969	397.8	83.421	88.701
1965	431.6	17.606	428.6	81.442	86.957
1966	415.6	24.292	467.6	79.058	85.391
1967	576.0	18.807	473.0	87.774	87.610
1968	545.1	12.756	490.4	92.683	86.829
1969	512.4	17.032	502.8	98.983	91.230
1970	665.4	21.010	576.6	116.834	94.185
1971	680.5	24.345	526.6	116.769	100.966
1972	625.2	32.592	559.7	117.269	104.954
1973	682.5	29.601	630.9	113.586	93.541
1974	743.0	18.987	685.3	108.057	104.110
1975	748.6	16.220	551.0	118.544	111.492
1976	773.2	12.041	643.9	123.087	114.470
1977	780.8	.	624.7	.	.

FIGURE 8-1
NICKEL PRIMARY PRODUCTION
NORTH AMERICA (▲), OCEANIA (■)
CENTRALLY PLANNED (●)
(thousands of metric tons)

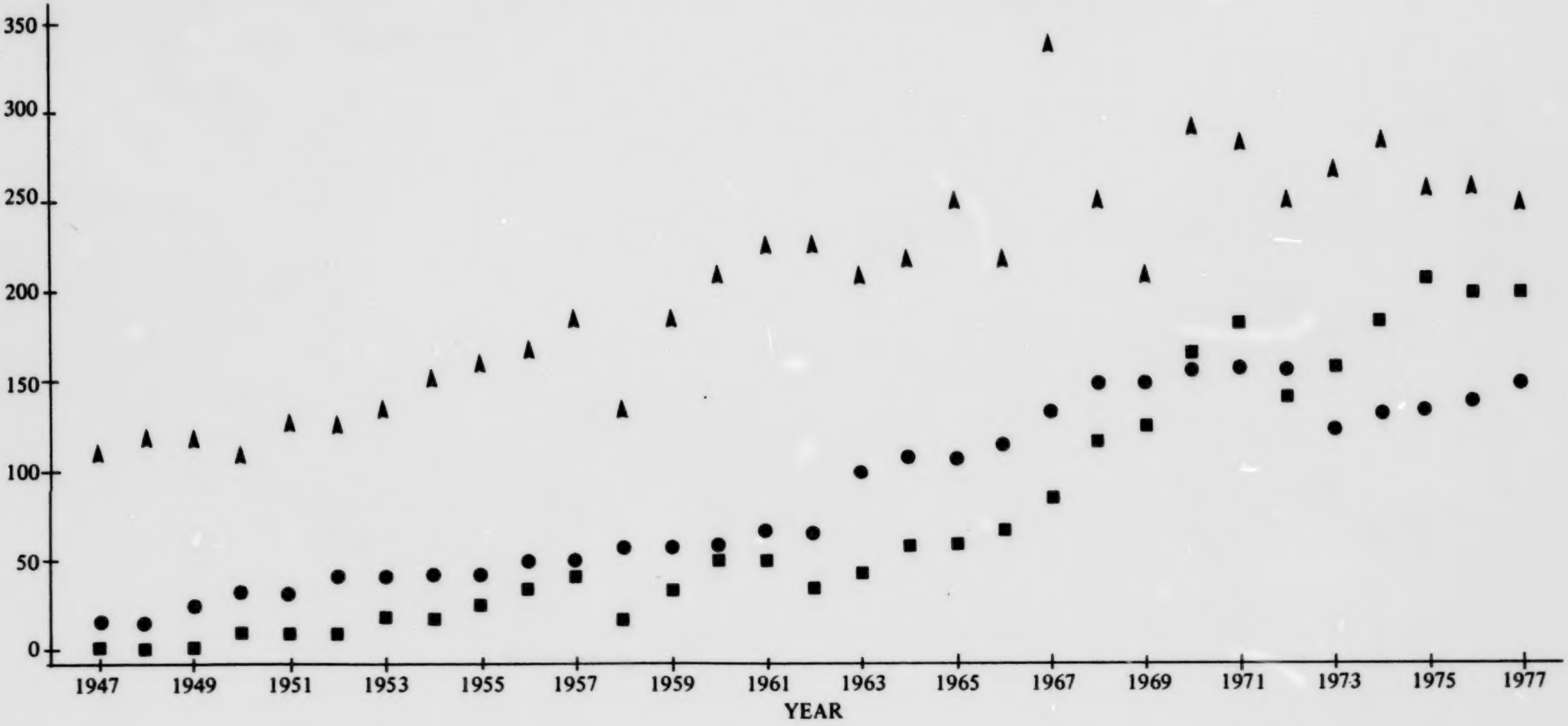


FIGURE 8-2
NICKEL PRIMARY PRODUCTION
SOUTH AMERICA (▲), ASIA (■)
AFRICA (●) EUROPE (□)
(thousands of metric tons)

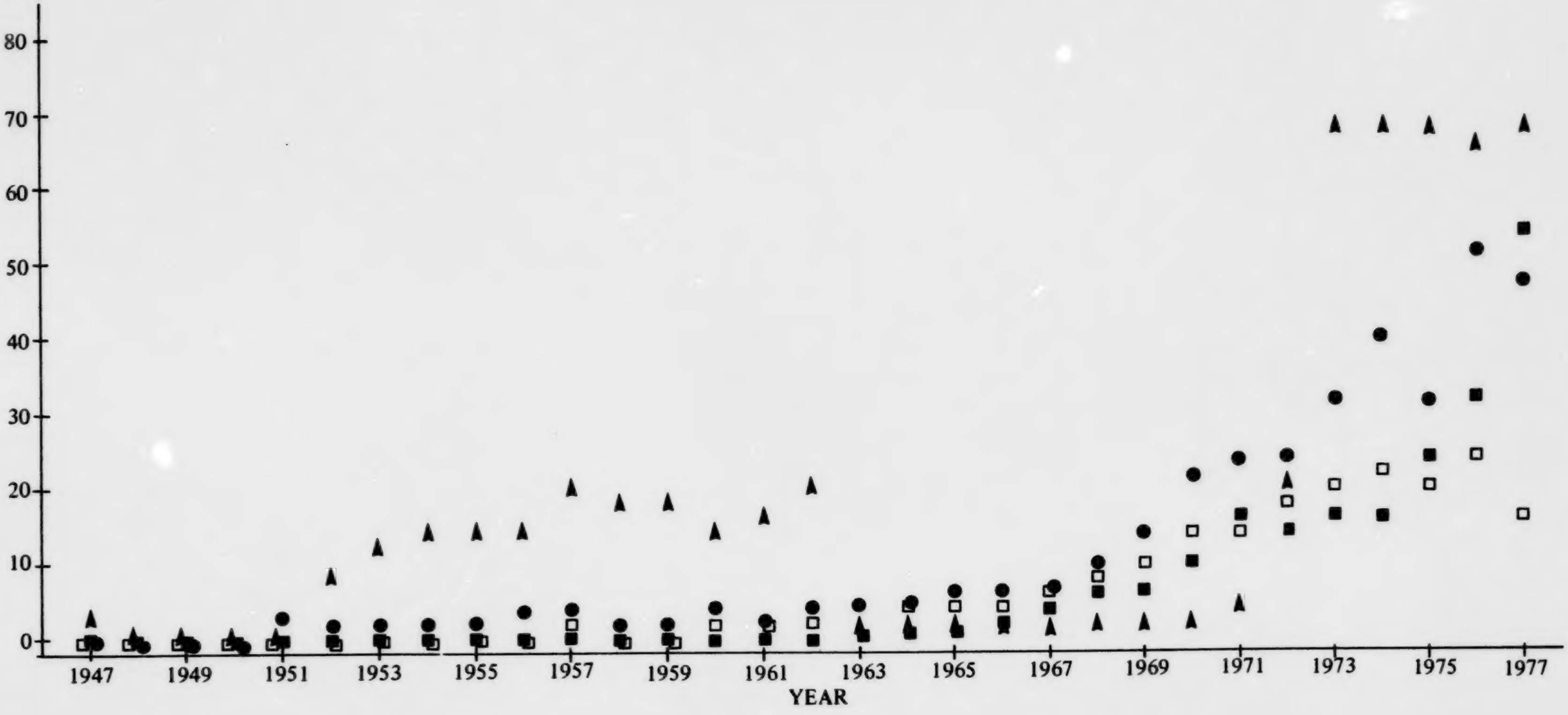


FIGURE 8-3
NICKEL CONSUMPTION
NORTH AMERICA (▲), EUROPE (■)
CENTRALLY PLANNED (●)
(thousands of metric tons)

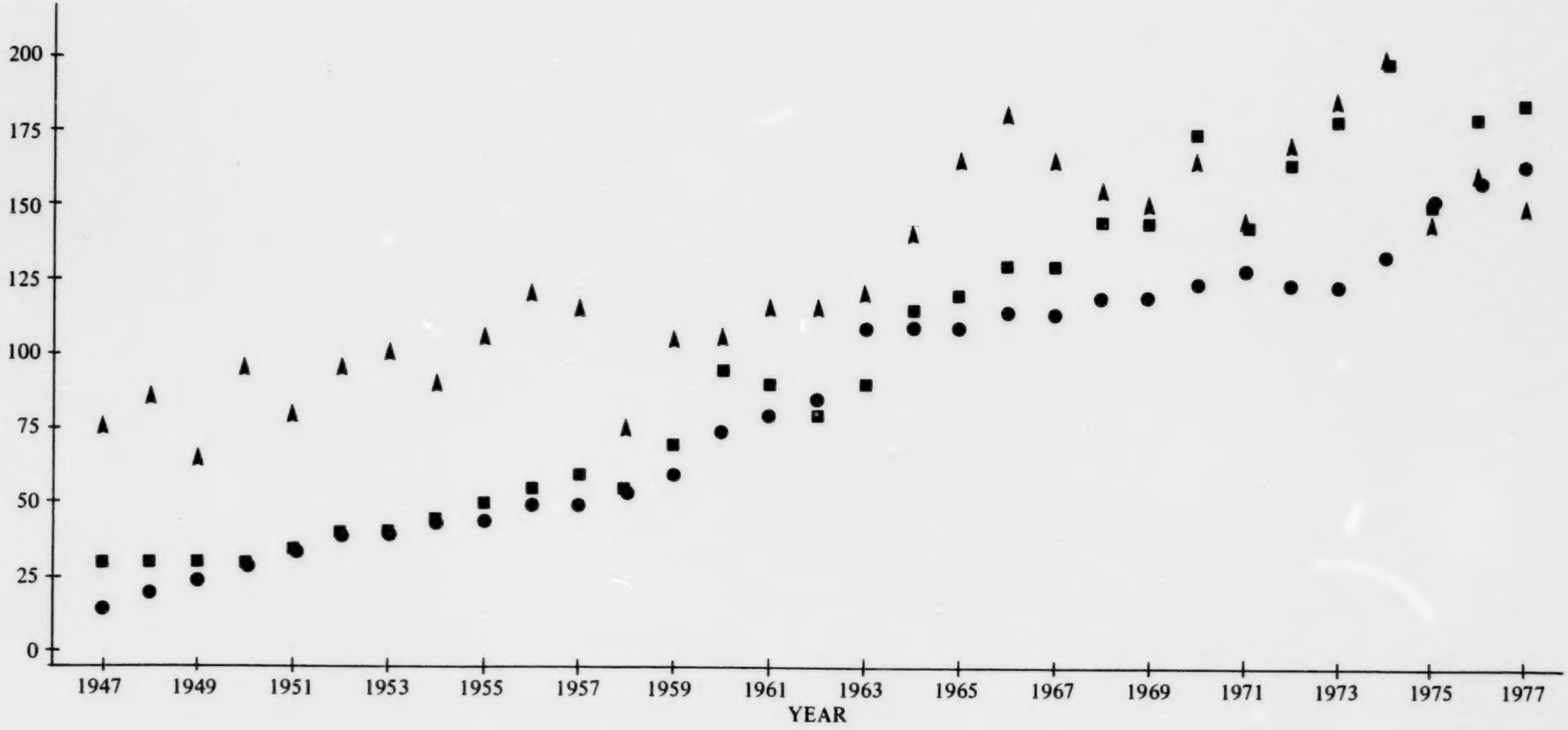


FIGURE 8-4
NICKEL CONSUMPTION: ASIA (▲)
(thousands of metric tons)

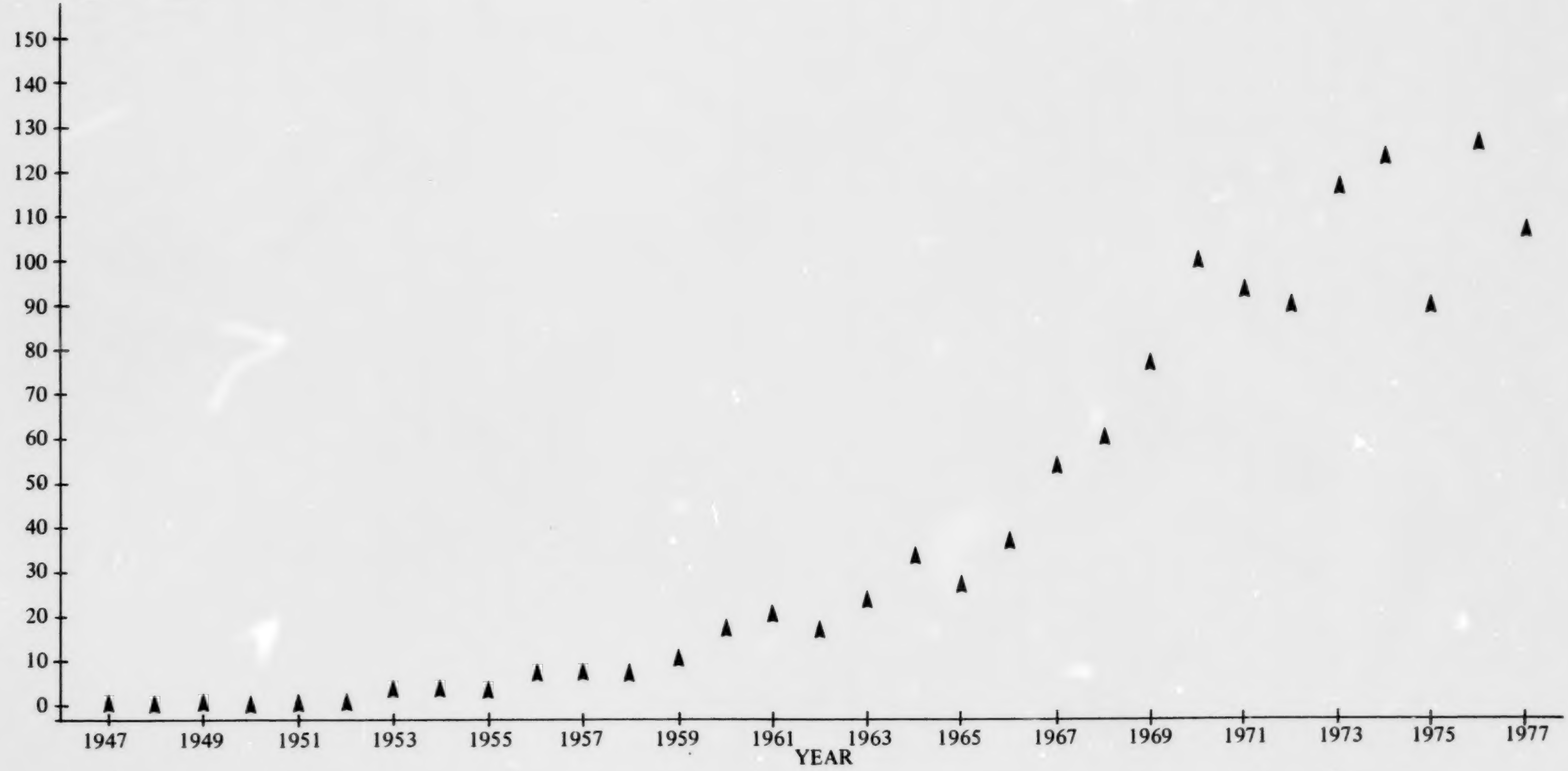
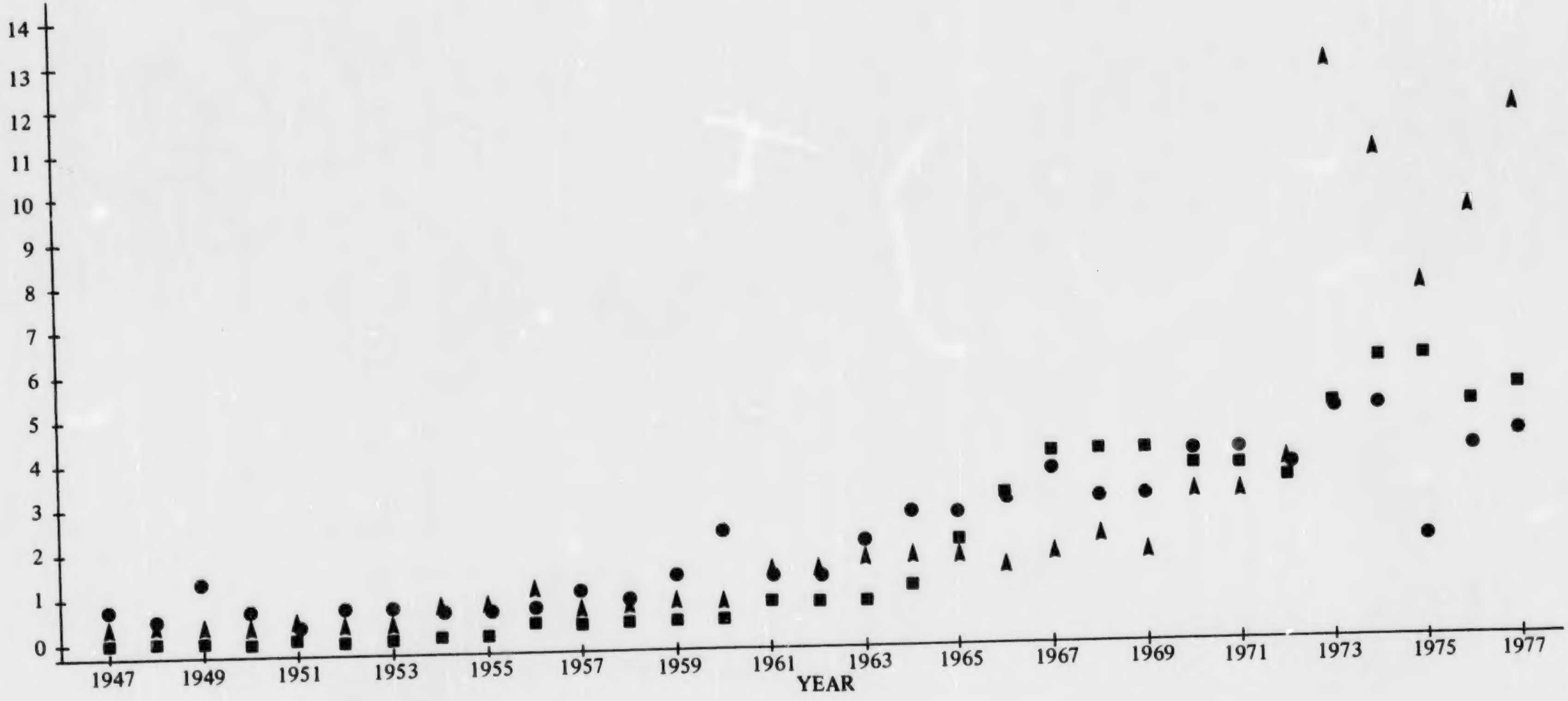


FIGURE 8-5
NICKEL CONSUMPTION
SOUTH AMERICA (▲), AFRICA (■)
OCEANIA (●)
(thousands of metric tons)



8.3 EMPIRICAL RESULTS

Using the data described in Chapter 5 and the preceding section, we estimated the equations developed in Chapter 3. The results of these estimations are presented in Tables 8-5 through 8-8. Following is a brief discussion of these estimates.

8.3.a PRIMARY PRODUCTION

From numerous sources, we were made aware of the assertion that the nickel market is one in which a price leader has historically existed. Since the purported price leader is in the North American group, it might prove interesting to compare these results with those of the other country groups to determine if such an assertion is supported empirically.[2] First, it should be noted that both current price and the determinants of expected price (i.e., $P_{M,t-1}$ and $ROWS_{M,t-1}$) are less significant in North America than in the other country groups. Such evidence is consistent with a situation in which the dominant firm [country group] sets price and the other firms behave as price takers. In the context of dominant firm price leadership, we would expect only the dominant firm to be concerned with total (i.e., world) demand or consumption. The estimated magnitudes of the parameters for current and lagged consumption would tend to indicate such behavior. That is, the magnitudes of θ_4 and θ_5 are much larger than those for all of the other country groups except Oceania (in which large deposits have been found). Thus, our estimates, while obviously not conclusive, are consistent with the existence of a price leader in North America. It is further interesting to note that expected demand (indicated by $D_{M,t-1}$) exerts a very significant effect; thus the dynamic nature of the production process is again supported.

For South America, current price proved unsatisfactory and was eliminated. In this context, the parameter θ_3 carried a sign opposite to that hypothesized but was statistically insignificant. The exclusion of current price suggests that the producers are unable to adjust substantially to unanticipated prices. Such an interpretation was supported by the substantial trend effect (i.e., $Q^S_{M,t-1}$) in this equation.

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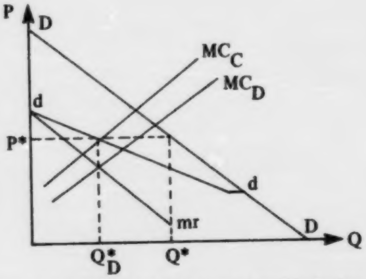
[2] To review briefly, a dominant firm price leader would set price to maximize his profit in his "protected" market. A graphical exposition will probably aid in the understanding of this idea. In the following figure, let MC_D be the marginal cost curve for the dominant firm and MC_C be the sum of the marginal cost curves for the other firms. The dominant firm sets price and allows the other firms to sell as much as they wish. In this way the dominant firm is left, from the market demand curve DD, with a "left-over" demand curve, dd. Associated with dd is a marginal revenue curve, mr. The dominant firm maximizes profit by equating his marginal cost curve with this left-over marginal revenue curve. Thus, he will set a price of P^* and produce Q^*_D. The other firms will, at this price, produce $Q^* - Q^*_D$; thus the market equilibrium is established.



The primary points of this discussion are the following: (1) The dominant firm is the only one interested in the market demand function. (The other firms take price as given.) (2) The non-dominant firms should be more sensitive to price changes than the dominant firm. Hence, empirically, we would expect the dominant firm to be more responsive to total demand (consumption) conditions, while the non-dominant firms would be expected to be more sensitive to price.

TABLE 8-5
ESTIMATED EQUATIONS
PRIMARY PRODUCTION
NICKEL
(T-STATISTICS IN PARENTHESES)

		North America 1952-1976	South America 1952-1976	Asia 1953-1976	Africa 1952-1976	Europe 1952-1976	Oceania 1952-1976	Centrally Planned 1952-1976
θ_0	Constant	-536.046 (-0.7542)	3.5136 (0.0174)	-21.6663 (-1.1821)	85.8054 (4.5632)	12.9107 (1.2577)	198.652 (0.8957)	-128.379 (-1.3868)
θ_1	$P_{M,t}$	2.1440 (0.7740)		0.1378 (1.2432)	0.3340 (2.8874)	0.1138 (2.2994)	2.8391 (1.9561)	0.8205 (1.2063)
θ_2	$P_{M,t-1}$	-0.8400 (-0.5349)	0.4237 (0.7243)	-0.0828 (-0.7419)	0.3163 (2.3567)	0.0210 (0.3351)	-0.4690 (-0.5444)	-1.1070 (-2.0839)
θ_3	$ROWS_{M,t-1}$	-0.5490 (-0.9192)	0.0043 (0.0587)	-0.0020 (-0.1345)	-0.0188 (-1.4301)	-0.0000 (-0.0161)	0.2237 (1.7667)	0.1287 (1.6040)
$\theta4$	$D_{M,t}$	0.2143 (0.8894)	0.0928 (1.1546)	-0.0165 (-1.1553)	0.0495 (3.4183)	0.0268 (2.9159)	0.3414 (1.3624)	0.1005 (1.0257)
θ_5	$D_{M,t-1}$	0.3496 (1.9889)	-0.0319 (-0.3328)	0.0022 (0.1652)	0.0364 (1.4617)	-0.0037 (-0.3533)	0.1452 (1.5028)	-0.0137 (-0.1855)
θ_6	t	11.2255 (0.7246)	-0.7306 (-0.1601)	0.3727 (0.8550)	-2.6074 (-5.4514)	-0.5123 (-2.0467)	-10.6321 (-1.8494)	2.3793 (1.1963)
θ_7	r_t	-27.4274 (-1.7050)	NA	NA	NA	-0.1084 (-0.2332)	12.5823 (1.3231)	NA
θ_8	r_{t-1}	21.5487 (0.9856)	NA	NA	NA	0.3058 (0.4551)	9.7655 (0.4758)	NA
θ_9	$\overset{t-2}{\Sigma} Q^S_{M,i}$	-0.0310 (-0.2683)	-0.1195 (-0.8336)	0.1173 (1.6653)	0.2736 (6.1019)	0.0252 (0.6481)	-0.0492 (-1.0557)	-0.0499 (-1.8801)
θ_{10}	$Q^S_{M,t-1}$	-0.3191 (-1.3744)	0.9513 (6.3520)	0.6637 (1.6290)	-0.4809 (-1.7839)	0.6059 (2.1608)	-0.1467 (-0.4527)	0.7002 (3.6632)
	R^2	0.8942	0.8516	0.9789	0.9909	0.9944	0.9822	0.9669
	SEE	23.5007	10.4052	1.6087	1.6479	0.7794	11.6190	9.8449

In the case of Asia, the signs of the parameters estimated for current world consumption, θ_4, and for previous extraction, θ_9, were not as predicted. We have no explanation for this anomoly. However, it should be noted that primary production in Asia appears to follow a trend modified by current price.

In Africa, the sign of the parameter for previous extraction was not as predicted. Examination of all parameters indicated that both expected and actual price and demand appear to be significant determinants of primary nickel production.

Turning to primary production in Europe, the sign of the parameter for previous production was not as predicted but the parameter estimate was statistically insignificant. Of particular interest in this equation were the very significant estimates for the parameters of current price and consumption. These parameters indicate substantial adjustment of production plans in Europe to altered market conditions.

Our estimated equation for Oceania carried all of the anticipated signs. Indeed, examination of these estimates indicates that primary production in Oceania behaves very much in the manner we posited in Chapter 3.

Finally, for the Centrally Planned Economies, expected price (i.e., $P_{M,t-1}$ and $ROWS_{M,t-1}$) clearly is a significant determinant and there is evidence of a substantial trend effect.

TABLE 8-6
ESTIMATED EQUATIONS
SECONDARY PRODUCTION
NICKEL
(T-STATISTICS IN PARENTHESES)
1952-1976

ϕ_0	Constant	-4.7355 (-0.6651)
ϕ_1^P	$P_{M,t}$	
ϕ_2^P	$P_{M,t-1}$	0.1414 (1.0547)
ϕ_3^P	$P_{M,t-2}$	
ϕ_1^D	$Q_{M,t}^D$	0.0188 (-0.1732)
ϕ_2^D	$Q_{M,t-1}^D$	-0.0053 (-0.1982)
ϕ_3^D	$Q_{M,t-2}^D$	-0.0175 (-0.6156)
ϕ_4^D	$Q_{M,t-3}^D$	
ϕ_S	$Q_{M,t-1}^{SCRAP}$	0.5675 (2.1271)
	R^2	0.6847
	SEE	4.5439

8.3.b SECONDARY PRODUCTION

As we noted previously, scrap data were available only from the United States. Our estimates using these data are presented in Table 8-6. Estimates using current price proved unsatisfactory and this variable was excluded. Although the sign of lagged price was not as predicted in our general model, the exclusion of current price puts this sign into question. An overall evaluation of these results indicates that secondary production is primarily determined by trend.

TABLE 8-7
ESTIMATED EQUATIONS
DEMAND
NICKEL
(T-STATISTICS IN PARENTHESES)

		North America 1952-1976	South America 1962-1976	Asia 1962-1976	Africa 1962-1976	Europe 1952-1976	Oceania 1952-1976	Centrally Planned 1952-1976
Ψ_0	Constant	-60.9557 (-1.6284)	-13.6289 (-2.0457)	-55.2326 (-1.0974)	-5.6538 (-1.7770)	-109.555 (-3.8180)	-4.5199 (-2.9994)	-1.1321 (-0.2499)
Ψ_1	$P_{M,t}-P_{R,t}$					-0.5936 (-1.0195)		-0.3983 (-1.2859)
Ψ_2	$P_{M,t-1}-P_{R,t-1}$	-0.3715 (-0.6197)				0.3463 (0.6189)		-0.0537 (-0.1911)
Ψ_3	$S_{M,t-1}-D_{M,t-1}$	-0.1417 (-1.4997)			-0.0068 (-1.8544)	-0.1564 (-2.4651)		-0.0058 (-0.1861)
Ψ_4	Y_t	326.385 (2.4820)	32.6026 (0.8391)	486.279 (2.9587)	2.3808 (0.2912)	656.184 (3.4234)	15.2643 (2.1459)	9.5325 (0.0963)
Ψ_5	Y_{t-1}	-104.639 (-0.6652)	-15.9706 (-0.3785)	-408.513 (-2.1637)	5.6490 (0.6970)	-363.461 (-1.5960)	-7.7967 (-1.0138)	23.0372 (0.2211)
Ψ_6	$\dot{K}_{t-1}$	118.352 (1.7264)		91.7363 (0.7488)	-5.5333 (-0.9343)	30.6500 (0.4069)	2.0027 (0.4796)	NA
Ψ_7	$\dot{M}_{t-1}$	0.6513 (0.3157)	NA	NA	NA	-0.8690 (-0.8332)	0.0272 (1.0172)	NA
Ψ_8	$Q^D_{M,t-1}$	-0.0001 (-0.0006)	0.2464 (0.8530)	0.3353 (0.7887)	0.7115 (4.0414)	-0.1446 (-0.5141)	0.0133 (0.0601)	0.8189 (7.0297)
	R^2	0.9013	0.7414	0.9247	0.9298	0.9726	0.8306	0.9820
	SEE	12.4285	2.2510	12.3505	0.5893	10.3081	0.6699	5.9106

8.3.c DEMAND

Our estimates for the demand functions are presented in Table 8-7. At the outset, it should be noted that current price (indicating the ability on the part of consumers to adjust to unanticipated price) proved satisfactory only for Europe and, surprisingly, the Centrally Planned Economies. Lagged prices proved to be unsatisfactory in the cases of South America, Asia, Africa, and Oceania. An overall evaluation indicates that the price effects are not very substantial in determining the demand for nickel.[3] Noting this, let us now examine the individual functions to see if we can isolate the primary determinants.

In the case of North America, the estimated parameters Ψ_3 and Ψ_8 carried signs opposite to those predicted and it is quite clear that the latter is statistically insignificant. Overall, it appears that the demand for nickel in North America is primarily determined by the demand for commodities that use nickel as an input and that there exists considerable adjustment to unanticipated demand [income] conditions.

For South America, none of the remaining determinants appeared to dominate. Although the overall equation was satisfactory, none of the individual parameters was significant.

In Asia, the demand for nickel appeared to be determined by the demand for final commodities (reflected by the income variable). Further, these estimates indicate that there exists considerable adjustment of nickel consumption to unanticipated income conditions.

In Africa, our estimates indicate that the trend in consumption dominates. While there is evidence of a price effect (see Ψ_3), without the price variables, the interpretation of this effect is not clear.

For Europe the estimated sign of the parameter for lagged consumption was the opposite of that predicted theoretically. The parameter was statistically insignificant. These estimates indicate that expected price and expected income (i.e., the demand for final output) are primary determinants of quantity demanded. Additionally, it appears that consumption plans can be altered substantially in response to unanticipated income conditions.

In the case of Oceania, only current income is shown to be a significant determinant. This implies that consumption decisions for nickel may be altered rapidly in response to existing demand for those final commodities that use nickel as an input.

Examination of our estimates for the Centrally Planned Economies indicates that consumption follows trend (see $Q^D_{M,t-1}$). Although there exists some evidence of adjustment to unanticipated price conditions, this effect appears slight in comparison with the trend variable.

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[3] If nickel is used as an input in the production of another product (e.g, steel) that is then used as an input in final production processes, such a result is clearly not unexpected.

8.3.d MARKET PRICE

Our estimated price equation is presented in Table 8-8. As we have noted in Chapters 3 and 5, we make no hypotheses concerning the signs of the parameters in this equation. It is. however. interesting to note the extremely significant parameter for the once lagged price of nickel. This indicates a strong trend relation in the price of nickel. However, this trend is apparently modified by past prices of raw steel and by the other variables.

TABLE 8-8
ESTIMATED EQUATIONS
PRICE
NICKEL
(T-STATISTICS IN PARENTHESES)
1952-1976

η_0	Constant	-14.4384 (-0.3742)
η_1	$P_{M,t-1}$	0.7411 (3.1621)
η_2	$P_{M,t-2}$	-0.0570 (-0.2443)
η_3	$S_{M,t-1}-D_{M,t-1}$	0.0040 (0.1079)
η_4	t	1.2517 (1.1965)
η_5	r_t	5.2895 (1.8431)
η_6	$\overset{t-2}{\Sigma}\, S_{M,i}$	-0.0018 (0.6237)
η_7	$WQ^{SCRAP}_{M,t-1}$	-0.0616 (0.1787)
η_8	$P_{R,t}$	
η_9	$P_{R,t-1}$	0.3479 (1.1001)
η_{10}	$P_{R,t-2}$	-0.8846 (-3.0241)
η_{11}	Y_t	-10.0084 (-0.2263)
	R^2	0.9719
	SEE	4.0389

- 9 -
ZINC

9.1 AN OVERVIEW OVER THE ZINC MARKET

Zinc market characteristics were discussed in detail in Mineral Policy Background Paper No. 3: *Towards a Zinc Policy for the Province of Ontario*, issued in December 1976. Some of this material was updated in a paper given at the 11th Commonwealth Mining and Metallurgical Congress in May 1978 in Hong Kong and published by the Institution of Mining and Metallurgy in the proceedings of that congress. Only a very brief summary of pertinent elements of the zinc market is therefore given in the following paragraphs.

Most of the world's zinc is produced from polymetallic sulphide ores by the electrolytic process. This process produces very pure metallic zinc. In these ores zinc occurs almost always as a co-product together with other metals, primarily copper, lead and nickel. Although not recognized as a metal, zinc was used very early in history as an alloying material in brass. The physical properties of zinc metal were described first by Paracelsus in the early 16th century.

The leading producing countries for zinc concentrate are Canada, the U.S.S.R., Peru, the United States, Mexico, Japan, the Irish Republic and Poland. These major producing countries each produced about or over a quarter million metric tons in 1980.

Major zinc uses are, in order of importance: galvanizing, die-casting and brass, with relatively minor applications in the chemical industry, mostly in the form of zinc oxide, for vulcanizing rubber, photocopying, pharmaceuticals and paint pigments.

In 1980 the western world consumption was approximately 5% lower than in 1979, largely due to declines in demand in the U.S. automobile and construction industries, where declines in zinc consumption on a national basis were in the 15% to 20% range. It is expected that demand overall will not continue to decline significantly in the U.S.A. in 1981-82. U.S. demand should begin to recover from then on but declines in consumption will still be felt in Britain and Japan. Mine production on the other hand is liable to increase with expansions of capacity in Australia and North and Central America beginning to come on stream.

Zinc is an internationally traded commodity. The significant fact is that among the major consuming blocks the United States, the European Economic Community and Japan are essentially importers. Most marketing of zinc concentrate is done through contracts between the concentrate producer and the refiner for the total refining of the concentrate. The metal is usually sold by the refiner on the concentrate producer's account, with producers rarely taking physical delivery of the metal.

Looking at the secular trends in real zinc prices and in zinc consumption, two things become apparent. The first is the amazing price stability: in terms of 1967 U.S. dollars (deflated by the wholesale price index) the zinc price for all of this century fluctuated around the 15¢/lb. range, the secular trend being horizontal or very slightly dipping. Characteristic of the annual movements are three major market peaks, one in 1915, one in 1952 and one from 1974 to 1976. All three of these major peaks were followed by very rapid downward corrections, even in the face of continuing increases in consumption. This may suggest that the zinc market is quite competitive, that zinc usage is subject to a considerable degree of substitution, particularly by aluminum and by plastics, and that the price peaks — whether the three major ones mentioned or some small ones in between — are attributable largely to very temporary supply constraints. This behaviour of the real zinc price has to be kept in mind, when evaluating the results of the simulations carried out later. The simulations performed are based on data ending in 1977, the first year of a beginning decline, that is they are dominated by many years of first gradual and then very rapid increases. A realistic projection then would have to be based on a simulation incorporating an external shock reflecting this decline and bringing the long term projections in line with the secular trend.

Secondly, when looking at the pattern of consumption, we see as we did in the case of nickel a pattern of relatively slow but stable growth from 1900 to the end of World War II, the major recessions being reflected by dips during the early 20's and early 30's. This is followed, starting at the bottom of the immediate post World War II dip, by a period of steep growth from 1945-1952 to the late 1960's-early 1970's. Then the pattern changes again to a flatter growth rate, from the early 70's to today, roughly parallel to that characteristic of the first half of the century. In fact over the first half of the century the consumption growth patterns for nickel and zinc are almost exactly parallel, with zinc consumption consistently exceeding nickel consumption in absolute terms by a factor of just over 10. After World War II however zinc consumption grew somewhat less steeply than nickel consumption. Comparing zinc consumption patterns with the patterns of copper consumption, there is an almost exact parallel from 1900 to the early 1930's — here the orders of magnitude are the same — with the slightly flatter growth trend in the case of zinc starting in 1932 rather than after World War II.

When comparing real zinc and real copper prices, the picture is again very similar. It is of interest though, that during the 60's the lag structure seems to change. While during the earlier part of the century the price peaks for zinc are usually slightly ahead of those for copper, by the 70's increasing prices for copper appear to lead increasing prices for zinc. No unambiguous pattern emerges from a visual comparison of real zinc and real nickel prices.

As mentioned earlier, the simulations for the zinc market probably have to be corrected by an external shock reflecting the end of the long third quarter century inflationary overexpansion and the return to the dominating secular trend. Overall we would expect, for at least the first part of the 1980's a flat or slightly declining trend in real zinc prices and a flattening out of the consumption pattern similar to that suggested for the case of nickel. In light of expected changing, that is easing, external constraints upon the industry in the United States and of the major expansions of zinc production capacity in many other parts of the world now under way or nearing completion, it cannot be ruled out that problems of excess capacity may occur prior to a potential resumption, in at least major parts of the western world during the mid to late 1980's, of solid non-inflationary growth.

9.2 DATA

As with the other minerals, data were required for primary production, secondary production, and consumption of zinc by country group as well as the world price of zinc and of the related commodity. We used as the related commodity, raw steel.[1] The majority of the data were collected from *Metal Statistics*. Following is a brief description of the data.

We used, for primary production, the reported data for mine production, expressed in thousand metric tons. The country specific data are presented in Table 9-1 and plotted in Figures 9-1 and 9-2. Aggregating over the country groups, world primary production of zinc was obtained and is reported in column 1 of Table 9-4.

For secondary production we used the data on scrap recovery, expressed in thousand metric tons. In the case of zinc, the data were such that only total world secondary production was usable.[2] These data are displayed in Table 9-2 and repeated in column 2 of Table 9-4.

Consumption of zinc is again expressed in thousand metric tons. These data for the individual country groups are displayed in Table 9-3 and plotted in Figures 9-3 through 9-5. Total world consumption of zinc (i.e., the sum of the country group data) is shown in column 3 of Table 9-4.

We defined the world price of zinc to be the U.S. price in cents per pound.[3] To convert this nominal price into a real price, it was deflated by the U.S. wholesale price index (1967=100). These data are displayed in column 4 of Table 9-4.

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[1] Although we make no explicit hypothesis, we expect zinc and raw steel to be complements rather than substitutes in the production of final commodities.

[2] Data were reported for the United States, Japan and western countries, total. We felt that, given these data, a total world secondary production function would be most appropriate.

[3] More specifically, we used the EMJ price for prime western zinc in East St. Louis.

The world price of steel was defined as the U.S. steel billet price in dollars per net ton. These values were obtained from *American Metal Market Statistics*. These values were also deflated by the U.S. wholesale price index (1967=100). These data are presented in column 5 of Table 9-4.

Table 9-1
Zinc: Primary Production
(thousands of metric tons)

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1947	767.0	325.0	28.8	81.5	193.4	179.1	193.6
1948	783.9	310.4	35.9	93.3	231.4	180.3	233.3
1949	799.6	279.1	45.0	109.7	268.5	182.7	257.0
1950	849.7	344.2	52.9	150.3	314.1	201.0	274.8
1951	927.5	334.0	78.6	176.4	350.3	192.3	304.4
1952	941.5	415.0	98.9	196.6	378.7	199.6	377.8
1953	861.1	415.2	111.9	239.7	388.5	243.2	464.9
1954	771.0	427.6	126.4	204.4	424.6	256.7	490.3
1955	860.0	492.6	128.9	205.8	446.9	260.7	501.9
1956	875.4	487.0	143.9	269.1	476.8	282.5	531.8
1957	857.7	469.5	157.6	277.0	492.9	296.3	554.8
1958	759.4	419.1	168.9	275.8	505.3	298.5	589.0
1959	745.1	462.1	167.8	239.6	506.7	280.0	623.2
1960	764.1	507.9	187.7	272.1	495.8	322.5	667.4
1961	798.7	500.4	193.7	257.8	515.2	316.2	668.4
1962	878.4	465.2	219.3	247.3	501.7	343.1	689.1
1963	909.9	480.9	232.3	246.3	510.0	357.1	701.9
1964	1142.5	514.2	253.5	273.7	499.5	350.1	759.9
1965	1300.1	535.3	262.2	297.7	466.5	354.8	831.6
1966	1394.0	559.8	303.8	259.2	495.6	375.3	898.0
1967	1506.7	605.5	322.9	244.6	526.8	407.0	977.9
1968	1532.1	590.0	340.4	256.3	587.8	422.4	1037.6
1969	1597.3	635.7	351.8	250.8	653.1	511.5	1113.1
1970	1620.3	682.7	371.6	249.1	641.5	488.7	1181.9
1971	1589.6	717.4	400.1	254.8	619.9	454.6	1185.0
1972	1562.6	791.7	403.3	245.1	637.1	508.8	1253.6
1973	1835.0	849.2	394.4	241.2	667.2	481.1	1367.0
1974	1738.5	828.4	421.5	260.1	729.7	457.1	1396.7
1975	1697.4	805.4	448.3	273.5	701.5	500.8	1477.5
1976	1628.0	912.5	466.9	245.0	765.5	468.6	1463.6
1977	1756.4	935.8	494.9	261.4	828.2	491.8	1527.5

Table 9-2
Zinc: Secondary Production
(thousands of metric tons)

Year	Scrap
1947	.
1948	557.7
1949	601.7
1950	699.4
1951	711.2
1952	675.9
1953	663.9
1954	673.8
1955	731.1
1956	697.4
1957	645.7
1958	606.3
1959	685.8
1960	722.3
1961	670.1
1962	678.6
1963	691.8
1964	762.5
1965	826.1
1966	855.8
1967	813.9
1968	882.5
1969	872.7
1970	887.1
1971	909.7
1972	962.1
1973	1004.9
1974	926.4
1975	794.0
1976	908.8
1977	869.8

Table 9-3
Zinc: Consumption
(thousands of metric tons)

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1947	791.3	22.3	70.3	10.6	550.3	39.4	185.8
1948	817.5	24.9	72.2	12.5	563.2	43.5	221.9
1949	716.4	32.8	77.8	11.9	602.4	45.5	234.5
1950	964.3	31.4	77.4	15.1	679.6	51.9	253.1
1951	898.6	44.5	91.8	15.2	676.1	48.6	298.3
1952	816.6	38.3	93.6	12.6	628.3	50.5	325.8
1953	937.1	37.9	109.4	13.1	680.1	57.1	358.3
1954	841.4	50.4	132.8	17.0	863.3	62.1	388.0
1955	1065.9	50.5	147.2	19.9	916.3	71.7	405.9
1956	966.0	57.6	155.7	24.3	883.7	70.5	443.2
1957	890.1	49.7	186.3	25.3	924.8	80.3	446.3
1958	831.3	67.0	208.9	23.7	941.2	74.5	517.6
1959	922.6	65.8	217.2	21.7	999.6	80.1	544.4
1960	838.9	78.0	271.3	30.2	1100.6	93.9	592.3
1961	895.4	79.3	339.2	25.4	1133.1	78.9	601.7
1962	977.4	81.8	342.5	29.3	1119.4	82.1	625.0
1963	1068.7	92.3	428.8	40.6	1148.7	92.5	665.9
1964	1182.4	105.3	479.7	48.1	1245.0	108.4	676.7
1965	1316.2	106.8	452.0	42.9	1240.1	99.0	725.9
1966	1382.5	119.8	513.4	50.7	1226.5	94.4	751.8
1967	1230.3	123.6	602.4	62.1	1214.6	108.2	850.9
1968	1335.2	136.3	696.5	54.2	1342.1	105.4	914.3
1969	1367.9	155.3	779.8	64.5	1505.4	120.0	978.4
1970	1181.0	164.3	825.9	76.6	1463.8	122.0	1043.1
1971	1251.3	176.6	836.3	77.6	1450.2	115.5	1103.9
1972	1421.9	200.6	959.0	72.1	1600.8	127.5	1156.3
1973	1517.2	239.8	1051.0	87.1	1753.8	143.0	1263.6
1974	1308.8	236.3	941.5	98.7	1701.8	140.6	1334.8
1975	998.7	210.3	778.8	81.5	1266.0	96.8	1356.0
1976	1173.8	238.9	988.3	111.4	1468.0	108.2	1409.4
1977	1132.2	251.8	1008.3	100.7	1479.7	102.6	1452.2

Table 9-4
Zinc
World Primary and Secondary Productions
World Consumption and Prices

Year	Primary Production	Secondary Production	Consumption	Real Prices Zinc	Real Prices Steel
1947	1768.4	.	1670.0	13.7255	56.536
1948	1868.5	667.7	1755.7	16.4118	58.128
1949	1941.6	601.7	1721.3	15.4307	66.112
1950	2187.0	699.4	2072.8	16.9511	65.098
1951	2363.5	711.2	2073.1	19.7585	61.471
1952	2608.1	675.9	1965.7	18.1998	64.673
1953	2724.5	663.9	2193.0	12.4199	69.348
1954	2701.0	673.8	2355.0	12.1929	71.895
1955	2896.8	731.1	2677.4	14.0080	75.399
1956	3066.5	697.4	2601.0	14.8776	77.960
1957	3105.8	645.7	2602.8	12.2176	81.190
1958	3016.0	606.3	2664.2	10.8975	83.002
1959	3024.5	685.8	2851.4	12.0759	84.388
1960	3217.5	722.3	3005.2	13.6417	84.299
1961	3250.4	670.1	3153.0	12.2138	84.656
1962	3344.1	678.5	3257.5	12.2627	84.388
1963	3438.4	691.8	3537.5	12.6952	85.735
1964	3793.4	762.5	3845.6	14.3273	88.701
1965	4048.2	826.1	3982.9	15.0104	86.957
1966	4285.7	855.8	4139.1	14.5291	85.391
1967	4591.4	813.9	4192.1	13.8430	87.610
1968	4766.6	882.5	4584.0	13.1707	86.829
1969	5113.3	872.7	4971.3	13.7089	91.230
1970	5235.8	887.1	4876.7	13.8759	94.185
1971	5221.4	909.7	5011.4	14.1598	100.966
1972	5402.2	962.1	5538.2	14.9093	104.954
1973	5835.1	1004.9	6055.5	15.3363	93.541
1974	5832.0	926.4	5762.5	22.4516	104.110
1975	5904.4	794.0	4788.1	22.2750	111.492
1976	5950.1	908.8	5498.0	20.2240	114.470
1977	6296.0	869.8	5527.5	.	.

FIGURE 9-1
ZINC PRIMARY PRODUCTION
NORTH AMERICA (▲), EUROPE (■)
CENTRALLY PLANNED (●)
(thousands of metric tons)

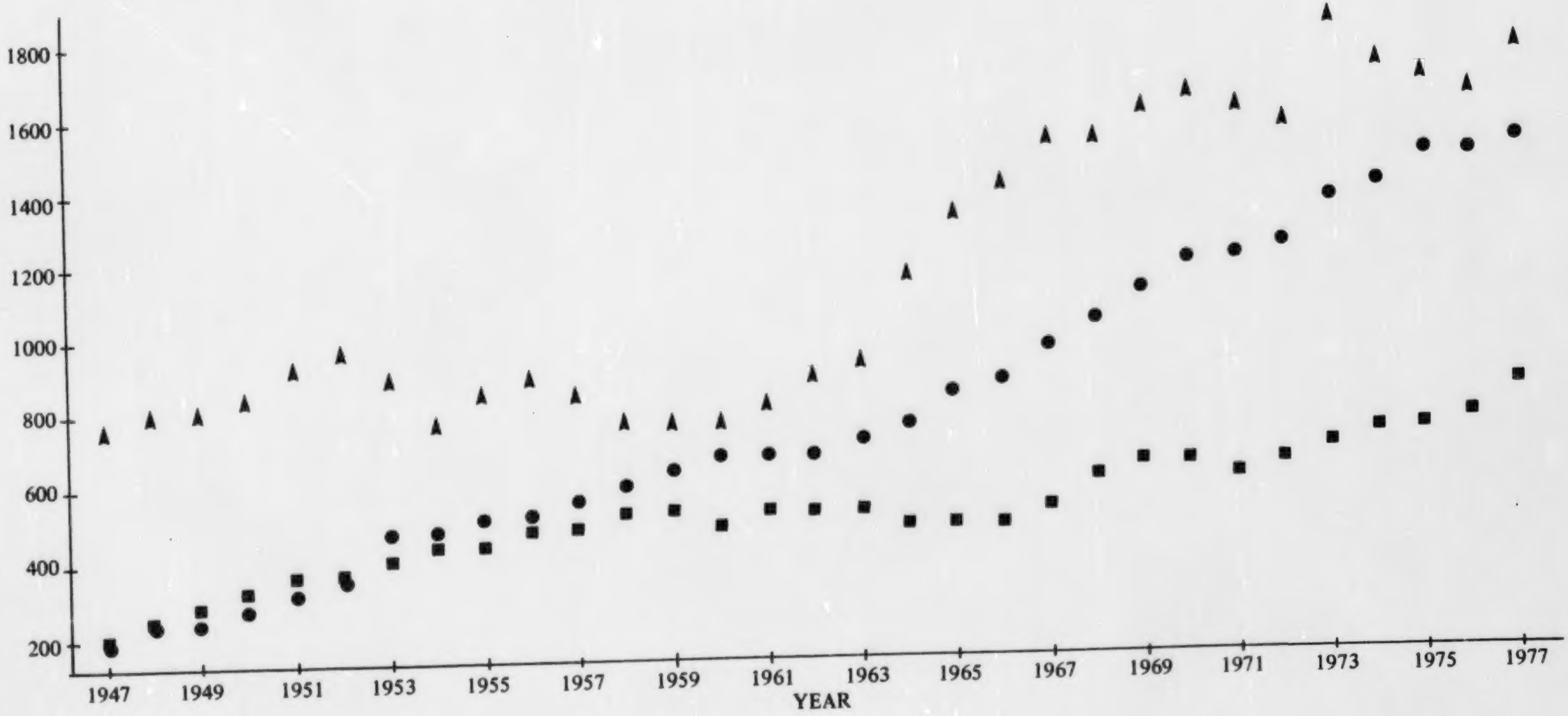

FIGURE 9-2
ZINC PRIMARY PRODUCTION
SOUTH AMERICA (▲), ASIA (■)
AFRICA (●), OCEANIA (□)
(thousands of metric tons)

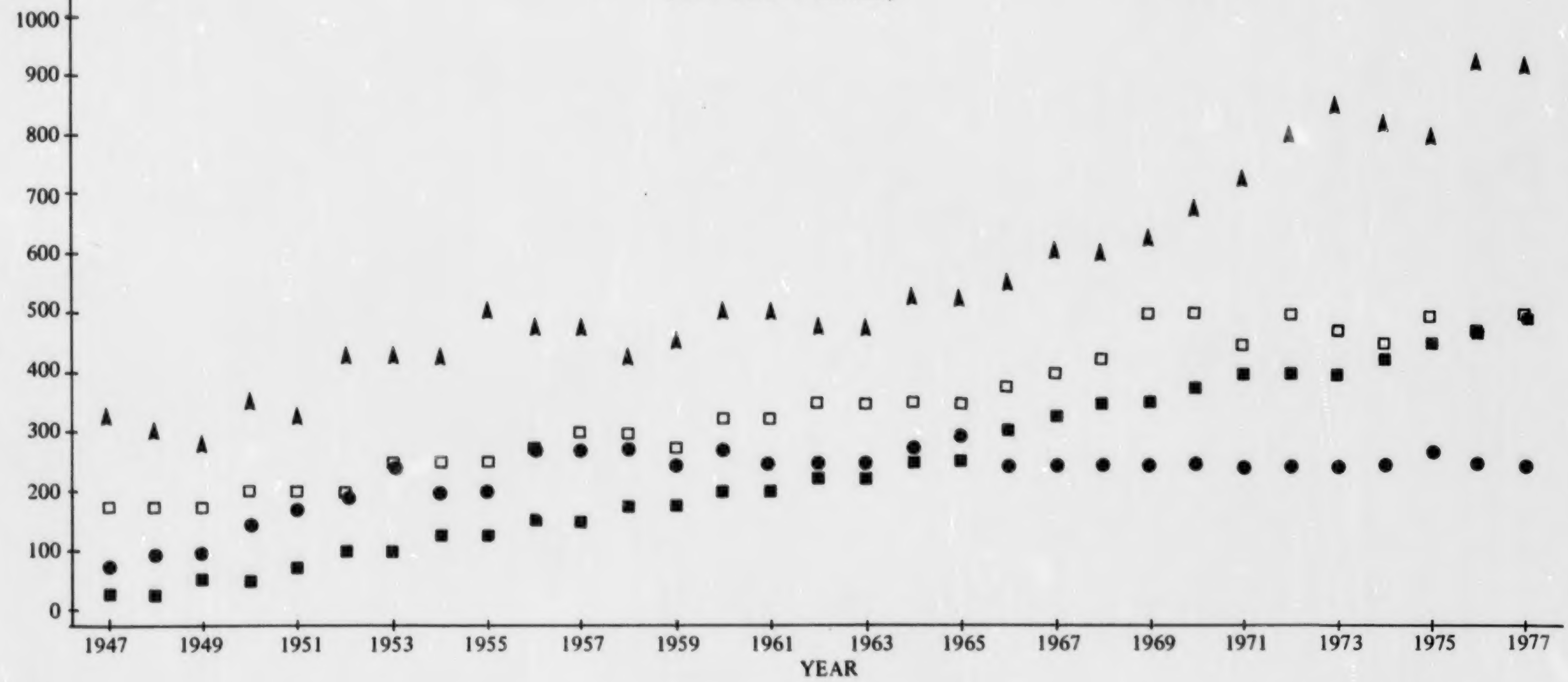

FIGURE 9-3
ZINC CONSUMPTION
NORTH AMERICA (▲), EUROPE (■)
CENTRALLY PLANNED (●)
(thousands of metric tons)

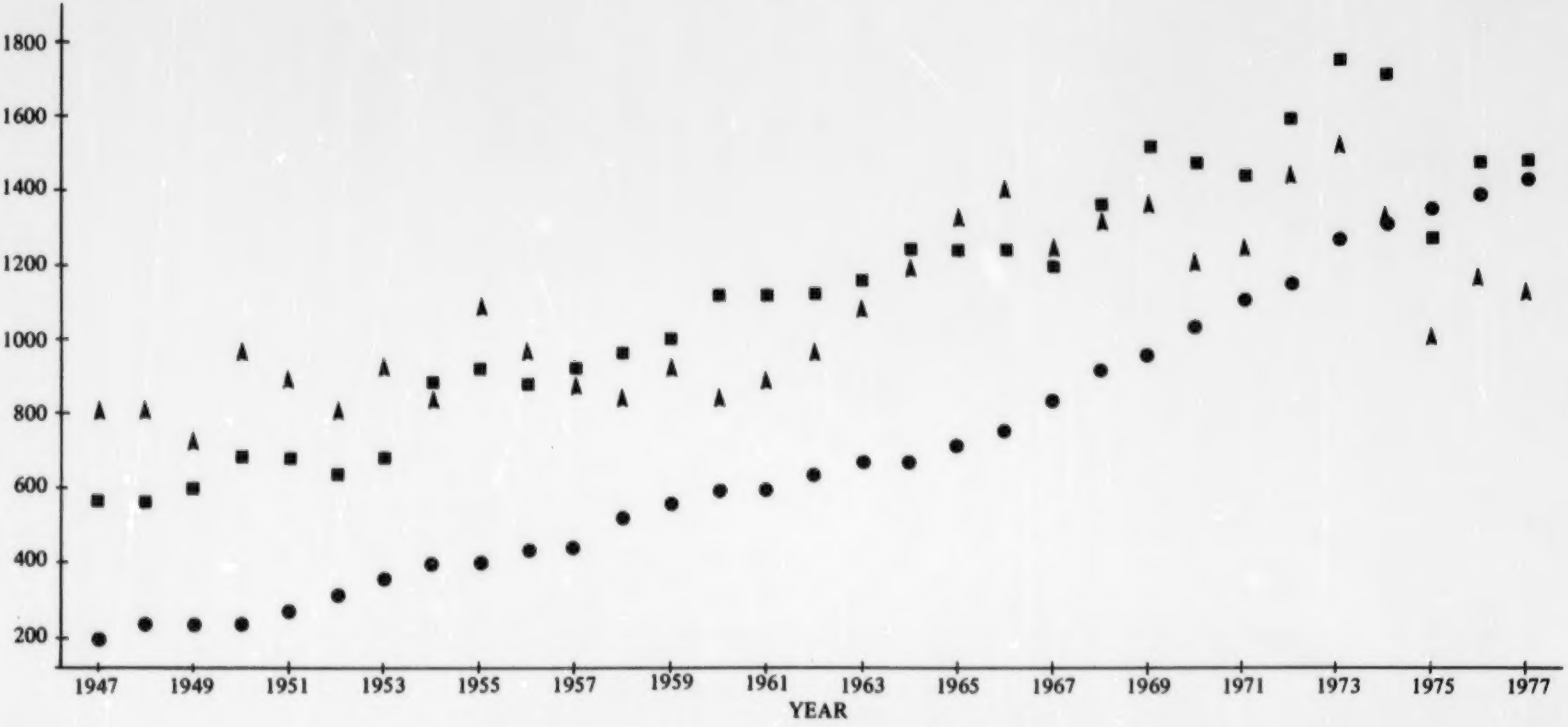

FIGURE 9-4
ZINC CONSUMPTION
SOUTH AMERICA (▲), AFRICA (■)
OCEANIA (●)
(thousands of metric tons)

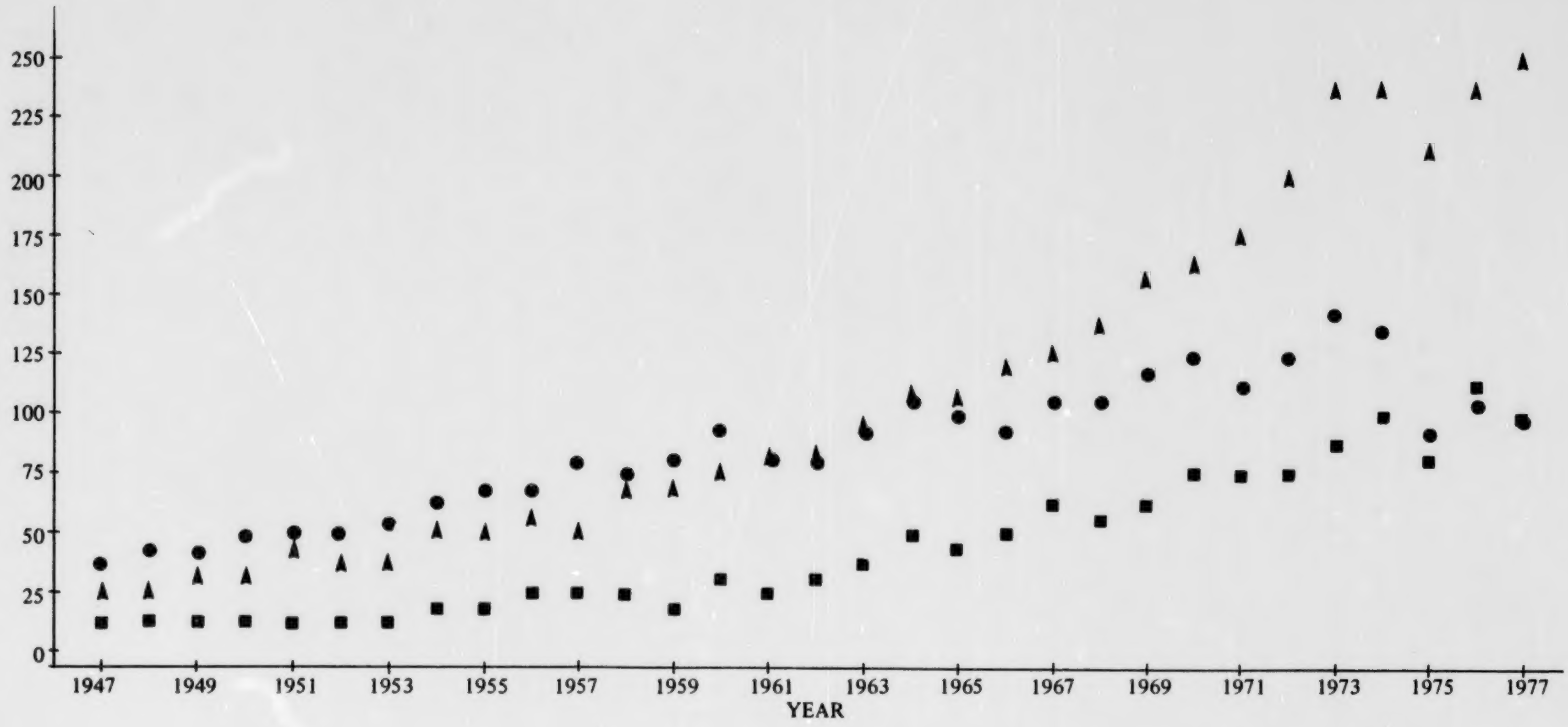

FIGURE 9-5
ZINC CONSUMPTION: ASIA (▲)
(thousands of metric tons)

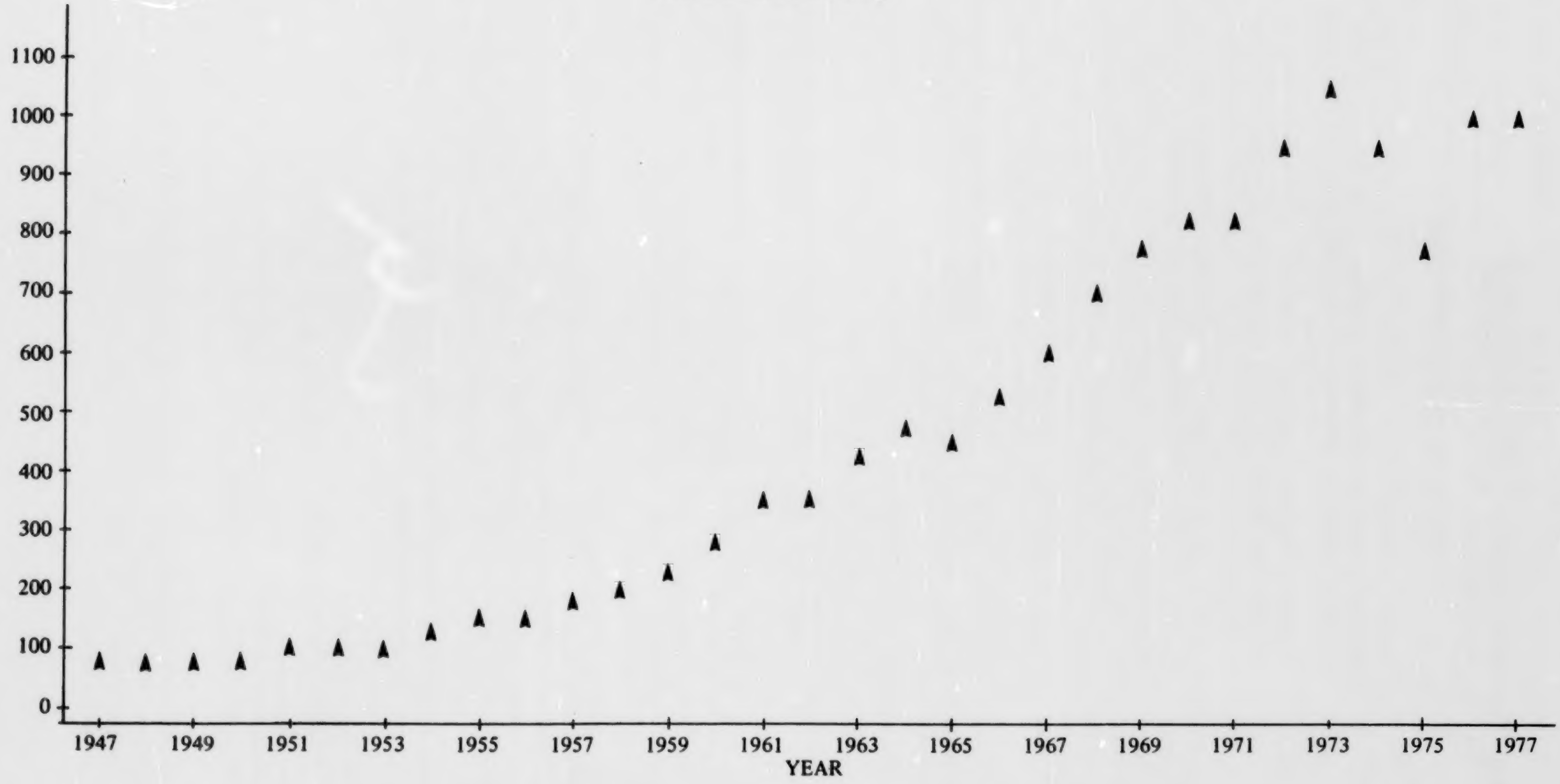

TABLE 9-5
ESTIMATED EQUATIONS
PRIMARY PRODUCTION
ZINC
(T-STATISTICS IN PARENTHESES)

		North America 1952-1976	South America 1952-1976	Asia 1952-1976	Africa 1952-1976	Europe 1952-1976	Oceania 1952-1976	Centrally Planned 1952-1976
θ_0	Constant	-2290.86 (-2.1425)	2384.92 (2.8506)	-698.097 (-3.6228)	-4564.43 (-2.8250)	-2231.52 (-0.5798)	-1331.89 (-1.0722)	-250.630 (-0.6096)
θ_1	$P_{M,t}$			5.1231 (2.1903)	6.0170* (1.8573)	9.1588 (0.8900)		
θ_2	$P_{M,t-1}$	35.9861 (2.9115)	0.5355 (0.1543)	0.7384 (0.4971)	-2.5143* (-0.8225)	-3.0990 (-0.7036)	5.2670 (0.7472)	6.6680 (1.2138)
θ_3	$ROWS_{M,t-1}$	0.1033 (0.4149)	0.0059 (0.1620)	0.0583 (2.8580)	0.0053 (0.1523)	0.0721 (1.4836)	0.0461 (0.9276)	0.0358 (0.5490)
θ_4	$D_{M,t}$	0.3176 (3.1907)	0.0868 (3.5731)	0.0036 (0.3175)	0.0192† (0.9625)	0.0596 (0.6305)	0.0622 (0.7560)	0.0463 (1.5490)
θ_5	$D_{M,t-1}$	0.0921 (1.4204)	-0.0197 (-0.7651)	-0.0242 (-1.8574)	-0.0445† (-1.0409)	-0.1097 (1.9524)	0.0413 (0.9586)	0.0378 (1.5814)
θ_6	t	25.9613 (1.5750)	-45.0289 (-2.9369)	10.5899 (3.4600)	93.8334 (2.9165)	41.5993 (0.6049)	21.2747 (1.2186)	1.5683 0.2497)
θ_7	r_t	1.0545 (0.0231)	NA	NA	NA	18.8497 (1.1159)	-27.3258 (-1.7113)	NA
θ_8	r_{t-1}	6.5047 (0.1062)	NA	NA	NA	5.0576 (0.1347)	49.0637 (1.1783)	NA
θ_9	$\Sigma^{t-2} Q^S_{M,i}$	-0.0749 (-1.9607)	0.0876 (2.7647)	-0.0489 (2.9099)	-0.3547 (-2.8914)	-0.0893 (-0.5738)	-0.1078 (-1.0065)	-0.0032 (-0.1476)
θ_{10}	$Q^S_{M,t-1}$	0.4614 (3.1343)	0.3139 (1.7090)	0.7071 (2.4231)	0.0284 (0.1147)	0.7367 (3.0912)	0.1803 (0.6437)	0.6321 (2.0630)
	R^2	0.9822	0.9787	0.9957	0.6638	0.9726	0.9568	0.9964
	SEE	65.7841	26.5206	9.4423	16.6421	22.6878	24.8486	24.7898

*Respectively $P_{M,t-1}$ and $P_{M,t-2}$
†Respectively $D_{M,t-1}$ and $D_{M,t-2}$

9.3 EMPIRICAL RESULTS

Using the data described in the preceding section as well as that discussed in Chapter 5, our econometric equations were estimated. The results of these estimations are displayed in Tables 9-5 through 9-8. Following is a brief overview of these results.

9.3.a PRIMARY PRODUCTION

In our estimations of primary production in North America, the current price variable proved unsatisfactory and was excluded. With current price eliminated, the parameter for once lagged price was estimated to be positive as predicted. The parameter for lagged primary production from the rest of the world carried a sign opposite to that predicted but this parameter was not statistically significant. As discussed earlier, the exclusion of current price suggests that the producers are unable to adjust their production schedule to unanticipated price conditions (or, that such adjustment is prohibitively expensive). The remainder of the parameters carried the hypothesized signs and were in general statistically significant.

For South America, current price again proved unsatisfactory. After elimination of this variable, the sign of the parameter for lagged price was as predicted. The sign for the parameter of lagged production in the rest of the world was opposite of that hypothesized but was statistically insignificant. The only troublesome parameter was that for preceding primary production (i.e., θ_9). It was opposite of that predicted and was statistically significant. Overall, primary production in South America appears to be heavily influenced by trend (see t and $Q^S_{M,t-1}$) modified by current consumption conditions.

The estimated primary production equation for Asia proved very satisfactory. Not only did all of the estimated parameters have the predicted signs but also the levels of significance for these parameters were high. In contrast with the estimated equation for North America, it would appear that Asian production is more responsive to unanticipated price than to unanticipated demand conditions.

In the case of Africa, both current price and current consumption proved unsatisfactory and were eliminated. Furthermore, our estimations indicated that the best results were obtained including twice lagged price and consumption. In this context, only lagged primary production from the rest of the world carried a sign other than as predicted and this parameter was again statistically insignificant. Examination of the estimates suggests that, while expected price is a significant determinant, African production exhibits a strong trend relation.

The estimated parameters for the primary supply function in Europe all carried the predicted signs. It would appear from these estimates that adjustment to unanticipated market conditions is not, however, substantial. Instead, these estimates suggest a long run trend in production based on expected consumption and, to a lesser extent, price.

For primary production in Oceania, current price proved unsatisfactory and was excluded. As occurred previously with this form, the sign of the parameter for lagged primary production in the rest of the world was opposite to that predicted and was insignificant. The signs of the remaining parameters were as predicted. However, given the levels of significance, the primary determinants of primary production in Oceania are not evident.

In the case of the Centrally Planned Economies, current price was again excluded. After excluding this variable, the parameter for primary production in the rest of the world again had a sign opposite to that predicted and was statistically insignificant. While the effects of expected price as well as expected and actual consumption are evident, it should be noted that there exists a strong trend effect ($Q^S_{M,t-1}$).

TABLE 9-6
ESTIMATED EQUATIONS
SECONDARY PRODUCTION
ZINC
(T-STATISTICS IN PARENTHESES)
1952-1976

ϕ_0	Constant	168.399 (2.0675)
ϕ_1^P	$P_{M,t}$	3.2410 (0.6926)
ϕ_2^P	$P_{M,t-1}$	0.6341 (0.1076)
ϕ_3^P	$P_{M,t-2}$	
ϕ_1^D	$Q^D_{M,t}$	0.1557 (6.3620)
ϕ_2^D	$Q^D_{M,t-1}$	-0.0942 (-1.6556)
ϕ_3^D	$Q^D_{M,t-2}$	-0.0229 (-0.5054)
ϕ_4^D	$Q^D_{M,t-3}$	
ϕ_S	$Q^{SCRAP}_{M,t-1}$	0.5037 (2.3366)
	R^2	0.9421
	SEE	31.7785

9.3.b SECONDARY PRODUCTION

As has been noted earlier, we employed scrap data only for the world as a whole. The estimates for the secondary production function are shown in Table 9-6. Of the estimated parameters, only that for lagged price was the opposite of our hypothesis and this parameter was statistically insignificant. The estimates indicated that both new and old scrap are significant in secondary production (i.e., both current and lagged consumption are significant). It appears that there exists a strong trend in secondary production modified slightly by price conditions.

9.3.c DEMAND

The estimated demand function for zinc for North America was, of all the country groups, the most satisfactory. All of the parameters carried the predicted signs. It appears from these estimates that the demand for final commodities that use zinc as an input (reflected by income) is the most significant determinant of quantity demanded.

In the case of South America, the current price variables proved unsatisfactory and were excluded. The remainder of the parameters carried the predicted signs. In this estimation, there exists evidence that quantity demanded of zinc is determined by expected price and income and that the consumers do adjust their plans in response to unanticipated income conditions.

For Asia, the current prices were again excluded. In this country group it appears that the demand for final commodities is again the primary determinant of quantity demanded. As has been mentioned in earlier chapters, this result is not surprising given the rapid growth in income in Asia over the sample period.

In our estimations of demand in Africa, both current and once lagged price proved unsatisfactory. Instead, we included the twice lagged values for the price variables. In this context, all of the parameters carried the hypothesized signs. Examination of the estimates indicated that quantity demanded is determined by expected price and that the consumers adjust their consumption plans in response to unanticipated income conditions.

Current price was excluded from the demand functions for both Europe and Oceania. In both of these, the signs for the lagged difference between world primary production and consumption as well as lagged consumption in the country group were the opposite of our hypothesized signs. (The latter parameters were, however, statistically insignificant.) In both of these country groups, our estimates indicated that quantity demanded is determined by expected price and income and that consumption plans are adjusted in response to unanticipated income conditions.

In the case of the Centrally Planned Economies, all of the price variables were found to be unsatisfactory. Our estimates indicated the existence of a significant trend that is altered by unanticipated income conditions.

9.3.d MARKET PRICE

Our estimated price equation is presented in Table 9-8. Again, since this equation is in essence a reduced form equation, we made no hypothesis concerning the signs of the parameters. The significant parameters for lagged price as well as time indicates the existence of trend. However, this trend is modified by the price of steel. Indeed, the negative sign for the parameter of the current price of steel supports the idea of complementarity between zinc and steel. Finally, it is interesting to note that our proxy variable for depletion ($\sum^{t-2} S_{Mi}$) exerts a significant effect. Our estimates indicate that the more of the resource that has been extracted in the past, the higher will be the price of the mineral.

TABLE 9-7
ESTIMATED EQUATIONS
DEMAND
ZINC
(T-STATISTICS IN PARENTHESES)

		North America 1952-1976	South America 1962-1976	Asia 1962-1976	Africa 1962-1976	Europe 1952-1976	Oceania 1952-1976	Centrally Planned 1952-1976
Ψ_0	Constant	203.834 (1.3602)	-72.9145 (-0.5113)	34.9991 (0.1269)	-194.291 (-3.3679)	-59.5058 (-0.5171)	-22.2901 (-1.2543)	-14.7821 (-1.0692)
Ψ_1	$P_{M,t}-P_{R,t}$	-2.8353 (-0.3701)						
Ψ_2	$P_{M,t-1}-P_{R,t-1}$	2.2260 (0.3404)	-2.4797 (-2.3199)	-1.8330 (-0.3780)	-1.2465* (-1.6342)	-5.0264 (-2.0939)	-0.6187 (-1.7011)	
Ψ_3	$S_{M,t-1}-D_{M,t-1}$	-0.0713 (-0.6499)	0.0171 (0.7677)	0.1028 (1.4690)	0.0222 (2.5788)	-0.2102 (-1.9634)	-0.0265 (-2.6537)	
Ψ_4	Y_t	4327.82 (3.6193)	384.042 (1.4755)	2547.14 (2.3088)	187.079 (1.6707)	4432.36 (4.4376)	230.119 (2.1440)	764.045 (2.4203)
Ψ_5	Y_{t-1}	-4333.05 (-3.6643)	-513.396 (-2.1095)	-2849.61 (-3.0499)	-11.4363 (-0.1023)	-3033.38 (-2.3102)	-123.653 (-1.0964)	-235.507 (-0.6620)
Ψ_6	$\dot{K}_{t-1}$	-102.535 (-0.1675)	-10.1757 (-0.1467)	869.824 (2.0286)	-86.4222 (-1.2222)	948.244 (1.7582)	1.4490 (0.0244)	NA
Ψ_7	$\dot{M}_{t-1}$	1.7915 (0.0829)	NA	NA	NA	-5.9214 (-1.0109)	0.3551 (0.9076)	NA
Ψ_8	$Q^D_{M,t-1}$	0.7264 (2.3855)	1.0113 (1.3574)	0.8940 (3.1203)	-0.0218 (-0.0595)	-0.2407 (-0.6204)	-0.1675 (-0.6058)	0.5233 (2.7687)
	R^2	0.8946	0.9821	0.9709	0.9410	0.9735	0.9001	0.9969
	SEE	85.4713	9.7926	51.5827	7.3524	57.7512	9.3241	20.2731

*Estimated coefficient for $P_{M,t-2}-P_{R,t-2}$

TABLE 9-8
ESTIMATED EQUATIONS
PRICE
ZINC
(T-STATISTICS IN PARENTHESES)
1952-1976

η_0	Constant	108.525 (4.3935)
η_1	$P_{M,t-1}$	0.5241 (3.6851)
η_2	$P_{M,t-2}$	-0.3482 (-1.6303)
η_3	$S_{M,t-1}-D_{M,t-1}$	-0.0022 (1.1850)
η_4	t	-1.6662 (-4.1352)
η_5	r_t	-1.1448 (-1.4761)
η_6	$\overset{t-2}{\Sigma} S_{M,i}$	0.0006 (4.1189)
η_7	$WQ^{SCRAP}_{M,t-1}$	-0.0557 (-0.6690)
η_8	$P_{R,t}$	
η_9	$P_{R,t-1}$	-0.2792 (-2.9008)
η_{10}	$P_{R,t-2}$	0.2112 (2.6249)
η_{11}	Y_t	-7.3347 (-0.6330)
	R^2	0.9183
	SEE	1.1397

- 10 -
SIMULATIONS

We now turn to the final objective of the research project, the provision of simulations of future values of production, consumption, and price for the four minerals explicitly considered — copper, aluminum, nickel, and zinc. In the most general terms, our simulation procedure involves the incorporation of projections of the future values for those variables that describe the world economic environment (e.g., income) into the econometric equations estimated. That is, we use as data projected values for the exogenous variables. It follows then that many alternative simulations for the variables under consideration may be obtained simply by altering the projections for the exogenous variables.

In this chapter we will provide some illustrative simulations. We begin with a very brief review of the simulation model itself. In this discussion, we again note that our simulation procedure involves both iterative and extrapolative techniques. Then, since an infinitely large number of simulations might be performed, we describe in Section 10.2 those that will actually be presented in this analysis. We must stress that these are only illustrative simulations. The reader can incorporate alternative assumptions into our model to obtain any other simulation of interest. Finally, in sections 10.3 through 10.6 we present respectively the simulations for copper, aluminum, nickel, and zinc. For the most part, we will present these simulations without comment and will leave both the interpretation and the selection of the most appropriate simulation results to the reader.

10.1 A REVIEW OF THE SIMULATION MODEL

The simulation technique that will be employed in our analysis incorporates elements of both an iterative and an extrapolative approach. It is iterative in the sense that the values of the variables under consideration in one period are used as data to generate simulated values for the next period. Thus, we will simulate values for period t and then use these simulated values to project the values in period t+1. It is extrapolative in the sense that we obtain the future values of the exogenous variables from historical trend and then use these extrapolated values as data to obtain simulations of the variables under consideration.

We begin with the primary supply, secondary supply, demand, and price equations estimated using data from the period 1952-1976. To review from Chapter 5, the form of these equations is as follows:

(1) World Price

$$\begin{aligned} P_{M,t} = {} & \eta_0 + \eta_1 P_{M,t-1} + \eta_2 P_{M,t-2} + \eta_3(S_{M,t-1} - D_{M,t-1}) + \eta_4 t \\ & + \eta_5 r_t + \eta_6 \Sigma^{t-2} S_{M,i} + \eta_7 WQ^{SCRAP}_{M,t-1} \\ & + \eta_8 P_{R,t} + \eta_9 P_{R,t-1} + \eta_{10} P_{R,t-2} + \eta_{11} Y_t \end{aligned} \tag{10-1}$$

(2) Demand in a Specific Country Group

$$\begin{aligned} Q^D_{M,t} = {} & \Psi_0 + \Psi_1(P_{M,t} - P_{R,t}) + \Psi_2(P_{M,t-1} - P_{R,t-1}) + \Psi_3(S_{M,t-1} - D_{M,t-1}) \\ & + \Psi_4 Y_t + \Psi_5 Y_{t-1} + \Psi_6 \dot{K}_{t-1} + \Psi_7 \dot{M}_{t-1} \\ & + \Psi_8 Q^D_{M,t-1} \end{aligned} \tag{10-2}$$

(3) World Demand

$$D_{M,t} = \Sigma^{j} Q^D_{M,t} \tag{10-3}$$

(4) Secondary Supply in a Specific Country Group

$$Q^{SCRAP}_{M,t} = \phi_0 + \phi^P_1 P_{M,t} + \phi^P_2 P_{M,t-1} + \phi^P_3 P_{M,t-2} + \phi^D_1 Q^D_{M,t} + \phi^D_2 Q^D_{M,t-1} + \phi^D_3 Q^D_{M,t-2} + \phi^D_4 Q^D_{M,t-3} + \phi_5 Q^{SCRAP}_{M,t-1} \qquad (10\text{-}4)$$

(5) World Secondary Production

$$WQ^{SCRAP}_{M,t} = \sum^{j} Q^{SCRAP}_{M,t} \qquad (10\text{-}5)$$

(6) Primary Supply in a Specific Country Group

$$Q^S_{M,t} = \theta_0 + \theta_1 P_{M,t} + \theta_2 P_{M,t-1} + \theta_3 \, ROWS_{M,t-1} + \theta_4 D_{M,t} + \theta_5 D_{M,t-1} + \theta_6 t + \theta_7 r_t + \theta_8 r_{t-1} + \theta_9 \sum^{t-2} Q^S_{M,i} + \theta_{10} Q^S_{M,t-1} \qquad (10\text{-}6)$$

(7) World Primary Production

$$S_{M,t} = \sum^{j} Q^S_{M,t} \qquad (10\text{-}7)$$

In these, the exogenous variables are the world price of the related commodity(P_R) and country group specific values for the interest rate (r), income (Y), the rate of capital accumulation ($\dot{K}$), and the rate of growth of the money supply ($\dot{M}$).

To perform our simulations, we first must obtain future values for the exogenous variables via extrapolation. For each of the exogenous variables, we calculated the mean historical rate of growth (straight line), $\bar{\dot{P}}_R$, $\bar{\dot{r}}$, $\bar{\dot{Y}}$, $\bar{\dot{K}}$, and $\bar{\dot{M}}$. Then, the t+1 values (e.g., 1977 values) will be extrapolated as

$$\tilde{P}_{R,t+1} = (1 + \bar{\dot{P}}_R) P_{R,t} \qquad (10\text{-}8)$$

$$\tilde{r}_{t+1} = (1 + \bar{\dot{r}}) r_t \qquad (10\text{-}9)$$

$$\tilde{Y}_{t+1} = (1 + \bar{\dot{Y}}) Y_t \qquad (10\text{-}10)$$

$$\tilde{\dot{K}}_{t+1} = \bar{\dot{K}} \qquad (10\text{-}11)$$

$$\tilde{\dot{M}}_{t+1} = \bar{\dot{M}} \qquad (10\text{-}12)$$

We then turn to our simulations of the variables under consideration. We first simulate the 1977 price using the extrapolated values for income, the interest rate, and the price of the related commodity in 1977 and existing values (i.e., 1975 and 1976 values) for the remainder of the variables in equation (10-1). Next, we use equation (10-2) to simulate consumption in 1977 in each of the country groups. In this, we must use our extrapolated income in 1977 and our *simulated* price in 1977 as data. Summing equation (10-2) over the country groups, we obtain simulated world consumption in 1977. Then, we use as data our simulated 1977 values for both price and consumption, as well as existing data for 1974, 1975, and 1976 in equation (10-4) to obtain a simulated value for scrap production in 1977 in each country group. Again summing equation (10-5) over the country groups, we obtain simulated world scrap production. To simulate primary production in 1977 in each country group, we employ existing 1976 data, the extrapolated value of the interest rate in 1977, and our simulations of price and world consumption in 1977 in equation (10-6). Then, we simulate world primary production in 1977 by summing equation (10-7) over the country groups.

To obtain simulations for 1978, the preceding steps will be repeated. However, there is one major difference. The 1978 simulations will require, as data, the 1977 values for the variables under consideration. Since these values are not available, we use as the existing 1977 values our simulated values for 1977. Again, the iterative feature of our model is that we simulate the values in period t and then use these simulations as data to simulate the t+1 values. The 1977 simulations will be used as data in the 1978 simulations; then, the 1978 simulations will also be used in the 1979 simulations, and so on.

As the reader will note, there exists one primary difference between our simulation model and our econometric analysis. In the econometric analysis we estimated each of the functions individually. However, in the simulation model, we combine all of the individual functions into a simultaneous equations model. Given this simultaneity, it is then possible that some of the functions, as estimated individually, would be inappropriate in the simulation model. That is, while a function independently may be satisfactory, it could yield perverse results once it feeds into and receives feedback from the other equations in the system. Therefore, it may be necessary to change the specifications of some of the individual functions.

In order to consider this difficulty, we will, prior to performing the simulations of future values, perform some in-sample predictions. We will accomplish the procedures outlined in this section to predict the values over a known period, using alternative specifications of some of the individual equations. Specifically, we will use the actual trends for the exogenous variables and the 1964 values for all of the variables to predict consumption, production and price for the period 1965-1977.[1] Then, by comparing our predicted values with the actual values, we can select those equations that provide the "best" simulation model, in the sense of providing those predicted values that most closely approximate the actual values.

10.2 SIMULATIONS TO BE PERFORMED

The simulation of future values described in the preceding section will be defined to be our "base" simulation. That is, all of the exogenous variables are assumed to follow historical trend. The historical growth rates for the country group data over the period 1962-1976 are as follows

	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
Income	0.028	0.030	0.047	0.018	0.033	0.028	0.056
Capital Stock	0.035	0.071	0.119	0.076	0.041	0.046	N/A
Interest Rate	0.050	N/A	N/A	N/A	0.039	0.048	N/A
Money Supply	0.052	N/A	N/A	N/A	0.087	0.074	N/A

It is, however, quite possible that the growth rates in the world or for specific country groups will differ from historical trend. Hence we might also wish to simulate the values for production, consumption, and price of the minerals under alternative assumptions about the rates of growth in these variables. Furthermore, it is quite possible that political conditions might alter production or consumption decisions. For instance, political instability might reduce production. Alternatively, a politically determined growth policy might necessitate increased mineral production to obtain the foreign exchange necessary to implement such a policy.

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[1] In essence, our in-sample prediction assumed that, in 1964, the growth rates for the exogenous variables over the period 1965-1977 were known with certainty. Using the 1964 values and the trends in the exogenous variables, we then simulated the values for production, consumption, and price for 1965 through 1977. It should be noted that none of the actual values for this period were used; however, the econometric equations used included this period as data.

In order to illustrate these features, we will, for each mineral, provide six alternative simulations. The assumptions are as follows:

Simulation #1 — The growth rates of income and capital accumulation will be one standard deviation above their historical means for all country groups.

Simulation #2 — The growth rates of income and capital accumulation will be one standard deviation below their historical means for all country groups.

Simulation #3 — The growth rates of income and capital accumulation in Asia will decline relative to those for Europe.

Simulation #4 — The growth rates of income and capital accumulation in both Asia and Europe will decline relative to those for North America.

Simulation #5 — As a result of non-market factors, primary production in Africa will be 5% below that level that would normally be predicted.

Simulation #6 — As a result of non-market factors, primary production in the Centrally Planned economies will be 5% above that level that would normally be simulated.

Hence, for each of the minerals under consideration, we will perform seven simulations. We will first present the "base" simulation (Simulation #0) and then present the six alternative simulations (Simulation #1-Simulation #6). It must be stressed that the simulations actually performed are only illustrative. The reader can incorporate any alternative assumptions into our model to obtain simulations of particular interest. We should again note that we will, for the most part, present the results of our simulations without comments. We leave the interpretation of these simulations as well as the selection of the most appropriate simulation to the reader.

Finally, we must note that all of the prices are expressed in 1967 U.S. dollars. We do this in order that the simulated future prices may be compared with the historical price series presented in Chapters 6-9. However, one may convert these prices into 1980 U.S. dollars simply by multiplying by 2.687.

10.3 COPPER

Upon performing in-sample predictions to finalize the simulation model, it became apparent that changes in the econometric equations would be required. Changes were made in some of the primary production and demand equations, as well as the price equation.

In the case of primary production, alternate equations were estimated for South America, Europe, Oceania, and the Centrally Planned Economies. (The primary production equations for North America, Asia, and Africa remain as presented in Table 6-5). The alternate equations are displayed in Table 10-1. In them, some of the explanatory variables were excluded.[2]

For demand, all of the country groups were reestimated except South America and the Centrally Planned Economies. These equations are presented in Table 10-2. As the reader will note, the primary difference between these estimates and those presented in Table 6-7 is the inclusion of a dummy variable that takes on a value of one in 1975 and 1976 and is zero otherwise. This additional variable was included to reflect the effect of the recession that occurred during this period.[3]

[2] The reader can compare these equations with those displayed in Table 6-5.

[3] Note in Table 6-3 the reduction in consumption that occurred during this period.

TABLE 1C-1
SIMULATION EQUATIONS
PRIMARY PRODUCTION
COPPER
(T-STATISTICS IN PARENTHESES)

		North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
θ_0	Constant		-1217.08 (-1.4625)			-12.5699 (-0.5564)	-48.0941 (-1.4994)	-239.1120 (-1.4600)
θ_1	$P_{M,t}$		0.3697 (0.2732)					
θ_2	$P_{M,t-1}$		-1.6621 (-1.3534)			0.0511 (0.1573)		1.1908 (2.7388)
θ_3	$ROWS_{M,t-1}$						0.0203 (0.4822)	
θ_4	$D_{M,t}$					0.0079 (1.0276)	0.0270 (1.3130)	
θ_5	$D_{M,t-1}$					-0.0027 (-0.2768)	-0.0299 (-1.1052)	
θ_6	t		32.8896 (1.8247)					4.3026 (1.3226)
θ_7	r_t		NA					NA
θ_8	r_{t-1}		NA					NA
θ_9	$\sum^{t-2} Q^S_{M,i}$		-0.0039 (-0.2295)			-0.0064 (-0.3577)		-0.0034 (-0.2731)
θ_{10}	$Q^S_{M,t-1}$		0.0578 (0.2034)			0.9336 (5.4435)	0.8542 (5.9984)	0.9898 (5.4467)
	R^2 SEE		0.9553 54.3194			0.9349 10.1128	0.9492 31.2419	0.9974 23.2695

TABLE 10-2
SIMULATION EQUATIONS
DEMAND
COPPER
(T-STATISTICS IN PARENTHESES)

		North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
Ψ_0	Constant	-263.821		-761.783	-79.0716	-100.226	-71.7731	
		(-1.0611)		(-1.7666)	(-3.1606)	(-0.5779)	(-2.9213)	
Ψ_1	$P_{M,t}-P_{R,t}$					-0.3387		
						(-0.0826)		
Ψ_2	$P_{M,t-1}-P_{R,t-1}$	-3.7993*		-2.1243	-0.2709	-10.9103	-0.3493	
		(-1.1372)		(-0.8245)	(-1.5337)	(-3.0553)	(-1.7803)	
Ψ_3	$S_{M,t-1}-D_{M,t-1}$	-0.2163			0.0060	-0.1660	-0.0131	
		(-1.1011)			(0.6998)	(-0.7104)	(-1.1697)	
Ψ_4	Y_t	5367.68		1457.17	1.2502		244.232	
		(4.4569)		(0.6360)	(0.0249)		(3.0878)	
Ψ_5	Y_{t-1}	-3488.63		448.012	117.147	2390.87	-99.0476	
		(-2.3346)		(0.1510)	(1.8436)	(4.0990)	(-0.9477)	
Ψ_6	$\dot{K}_{t-1}$	64.1942		1056.26	-93.5441	2330.78	106.307	
		(0.0678)		(1.3035)	(-2.5877)	(2.4035)	(2.2385)	
Ψ_7	$\dot{M}_{t-1}$	6.6888		NA	NA	-10.6219	-0.2543	
		(0.3343)				(-0.8430)	(-0.8355)	
Ψ_8	$Q^D_{M,t-1}$	0.1313		-0.3964	0.8346	0.0986	0.2847	
		(0.4154)		(-0.7341)	(6.3630)	(0.4272)	(1.2786)	
	Dummy Variable	-274.550		-427.871	-20.1522	-17.8323	-8.9993	
		(-1.5206)		(-1.5254)	(-2.8066)	(-0.1065)	(-1.0056)	
	R^2	0.9274		0.9318	0.9835	0.9627	0.9735	
	SEE	126.430		98.8461	3.7292	108.332	5.9820	

*Estimated coefficient for $P_{M,t-2}-P_{R,t-2}$

Our alternate price equation is as follows:

$$\begin{aligned}P_{M,t} = {} & \underset{(-0.1500)}{6.6342} + \underset{(1.0838)}{0.2523}P_{M,t-1} - \underset{(-0.0691)}{0.0162}P_{M,t-2} \\ & - \underset{(-2.2577)}{0.0292}(S_{M,t-1} + WQ^{SCRAP}_{M,t-1} - D_{M,t-1}) \\ & + \underset{(0.9633)}{0.9166}t + \underset{(0.0290)}{0.1234}r_t \\ & - \underset{(-0.8487)}{0.7767}P_{R,t} - \underset{(-0.3551)}{3.7819}\,\text{Dummy}\end{aligned}$$

$$R^2 = 0.6591 \qquad SEE = 8.7729$$

As the reader will note, substantial modifications were made. First, we changed our inventory variable from the difference between lagged primary production and consumption to the once lagged value of primary *plus* scrap production minus consumption. It was felt that this formulation more accurately reflects inventory behavior. Second, it was found that the price equation performed better (in the context of in-sample predictions) if the sum of past production, lagged price of aluminum, and current income were excluded. Finally, we again included a dummy variable to reflect the recessionary effects in 1975 and 1976.

Our in-sample predictions using this modified model are presented as Simulation #00. The reader can compare our simulated values with the actual values presented in Tables 6-1 through 6-4.

We then used this same model to simulate the future values of production, consumption and price. We used the growth rates for the country group variables as presented in section 10.2. Over the same period, the price of aluminum was declining at a rate of -0.004. We present the resulting simulations, as described in sections 10.1 and 10.2, as Simulation #0 through Simulation #6. As we noted earlier, we leave the interpretation and selection of the most appropriate simulation to the reader.

COPPER
SIMULATION # 00

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1965	1654.5	1963.3	875.4	134.2	254.1	598.5	1117.5	46.4	99.8	2129.2	124.5	104.2	849.2	1099.4
1966	1601.6	1960.9	884.9	132.0	269.3	627.2	1140.6	44.3	104.3	1902.3	134.5	95.1	926.6	1143.4
1967	1623.6	1909.7	915.3	143.8	287.7	694.0	1167.9	46.6	108.9	1927.4	152.3	93.5	1002.0	1183 8
1968	1712.6	1939.7	959.2	175.8	305.2	762.5	1209.2	53.4	114.5	2072.2	177.4	98.4	1067.8	1237.2
1969	1833.5	2038.8	997.9	203.0	322.6	823.8	1256.5	61.6	121.2	2202.9	204.8	105.6	1129.5	1303.1
1970	1941.5	2136.7	1026.9	220.5	341.6	879.3	1300.9	69.5	128.2	2274.5	231.1	111.9	1193.8	1373.9
1971	2029.4	2202.5	1052.5	235.3	363.0	938.8	1341.5	77.0	135.5	2323.2	256.6	117.0	1261.9	1445.6
1972	2114.2	2254.6	1080.7	254.1	386.3	1005.2	1381.9	85.0	143.1	2389.4	283.2	122.1	1331.0	1520.1
1973	2208.8	2317.3	1111.5	276.8	410.8	1076.3	1424.7	93.9	151.2	2477.8	311.6	128.1	1399.2	1600.2
1974	2312.1	2395.2	1142.1	300.5	436.6	1149.3	1469.5	103.1	159.9	2571.8	341.1	134.7	1466.7	1686.9
1975	2040.1	2204.9	1169.7	325.7	444.8	796.2	1476.0	92.1	163.2	2643.9	351.0	132.5	1534.6	1779.6
1976	2179.2	2191.4	1201.0	340.6	463.9	1051.7	1517.3	87.0	169.5	2723.9	378.6	134.4	1598.7	1867.8
1977	2277.1	2282.1	1241.0	381.4	480.8	1078.1	1565.0	85.9	176.4	2891.9	407.6	142.8	1655.2	1973.8

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1965	358.4	5.1	76.8	0.0	401.0	27.9	N/A
1966	368.9	10.8	79.1	0.0	422.7	28.2	N/A
1967	352.9	11.8	78.2	0.0	426.2	27.9	N/A
1968	348.4	12.7	82.7	0.0	407.5	29.7	N/A
1969	365.3	14.9	89.2	0.0	405.1	30.0	N/A
1970	388.6	18.8	95.0	0.0	419.9	30.9	N/A
1971	405.9	23.1	99.7	0.0	438.9	32.7	N/A
1972	417.6	27.1	104.6	0.0	454.0	34.7	N/A
1973	430.0	31.0	110.2	0.0	467.0	36.5	N/A
1974	446.4	35.2	116.6	0.0	482.2	38.3	N/A
1975	437.0	39.1	94.3	0.0	488.9	37.4	N/A
1976	416.3	41.6	84.9	0.0	484.6	35.4	N/A
1977	429.3	46.8	111.7	0.0	503.6	38.6	N/A
1978	453.2	51.8	117.8	0.0	526.2	40.4	N/A
1979	479.3	58.4	128.4	0.0	560.3	43.7	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1965	4975.1	6075.8	57.892	869.1
1966	5061.8	5905.2	55.747	909.7
1967	5257.7	5998.9	47.339	897.0
1968	5545.9	6339.0	43.661	881.1
1969	5866.0	6738.9	45.874	904.4
1970	6164.1	7066.3	49.170	953.2
1971	6440.5	7339.5	50.423	1000.3
1972	6720.5	7630.6	50.249	1038.0
1973	7017.9	7970.4	50.439	1074.8
1974	7328.1	8341.6	51.685	1118.7
1975	7179.4	7975.0	49.748	1096.8
1976	7508.2	8396.8	44.551	1062.9
1977	7803.1	8836.0	48.016	1130.0

COPPER
SIMULATION # 0

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	2615.6	2493.6	1314.1	460.0	554.1	1370.5	1700.6	86.9	205.9	3067.9	503.2	151.1	1819.4	2332.8
1981	2726.9	2552.7	1340.2	488.5	580.3	1472.8	1746.2	87.8	214.4	3159.6	530.7	157.9	1885.2	2459.3
1982	2866.5	2648.6	1369.9	522.3	608.1	1579.8	1797.4	90.4	223.9	3285.8	561.4	166.3	1948.6	2597.3
1983	3019.2	2768.8	1397.1	554.6	638.0	1687.5	1851.0	93.4	234.5	3404.7	593.4	175.4	2012.1	2746.2
1984	3170.7	2888.9	1420.7	584.5	670.6	1797.9	1904.3	96.3	245.8	3505.8	625.7	184.2	2077.7	2903.7
1985	3320.2	3001.0	1443.2	614.5	706.3	1914.1	1957.1	99.1	257.9	3603.5	658.7	192.9	2145.8	3069.1
1986	3474.1	3112.7	1466.4	646.7	744.8	2037.1	2011.0	102.1	270.9	3710.4	693.1	201.9	2215.4	3243.4
1987	3636.5	3231.9	1490.0	681.1	786.0	2166.3	2066.7	105.1	284.7	3826.8	728.9	211.3	2285.9	3428.1
1988	3806.8	3359.3	1513.2	716.4	830.1	2301.0	2124.1	108.0	299.4	3947.0	766.0	221.3	2357.7	3623.9
1989	3983.2	3492.0	1535.4	752.4	877.1	2441.5	2182.6	110.7	315.1	4067.9	804.3	231.5	2431.1	3830.9
1990	4165.1	3627.9	1556.9	789.4	927.2	2588.5	2242.3	113.1	331.6	4190.7	843.7	241.9	2506.3	4049.6
1991	4353.5	3767.8	1578.0	827.5	980.5	2742.5	2303.4	115.0	349.0	4317.4	884.4	252.7	2583.4	4280.6

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1980	482.4	66.4	136.4	0.0	572.1	42.9	N/A
1981	496.2	72.7	145.1	0.0	602.4	46.8	N/A
1982	517.2	78.9	155.2	0.0	628.5	50.1	N/A
1983	546.9	85.8	166.0	0.0	660.3	53.2	N/A
1984	579.6	93.6	176.9	0.0	697.7	56.9	N/A
1985	611.3	101.6	187.9	0.0	736.7	61.0	N/A
1986	642.7	109.8	199.3	0.0	775.5	65.2	N/A
1987	675.7	118.2	211.6	0.0	814.9	69.6	N/A
1988	711.3	127.0	224.5	0.0	856.5	74.2	N/A
1989	749.1	136.2	238.1	0.0	901.0	78.9	N/A
1990	788.4	145.8	252.2	0.0	947.7	84.0	N/A
1991	829.1	155.8	267.0	0.0	996.6	89.3	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1980	8713.0	9962.9	58.855	1300.3
1981	9023.9	10378.4	59.092	1363.2
1982	9375.9	10890.3	61.398	1429.8
1983	9745.2	11430.7	65.756	1512.3
1984	10115.6	11961.4	70.462	1604.6
1985	10489.3	12494.2	74.626	1698.5
1986	10875.6	13054.3	78.573	1792.5
1987	11278.7	13650.6	82.902	1889.9
1988	11697.3	14276.9	87.795	1993.5
1989	12128.7	14926.9	93.080	2103.2
1990	12573.2	15601.0	98.592	2218.1
1991	13032.2	16303.5	104.335	2337.8

COPPER
SIMULATION # 1

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	3180.2	3018.3	1307.4	529.2	600.5	1573.7	1771.2	88.3	223.7	3346.1	543.4	184.4	1828.8	2414.7
1981	3416.8	3187.2	1329.5	578.1	647.5	1721.7	1834.1	93.6	239.8	3496.8	583.7	198.4	1904.4	2584.0
1982	3597.4	3406.3	1357.1	635.9	699.8	1885.9	1905.7	102.7	258.3	3700.4	629.5	215.1	1979.4	2772.8
1983	4004.3	3668.3	1382.5	695.1	757.6	2054.3	1982.7	114.1	279.4	3908.7	679.0	233.4	2056.1	2981.4
1984	4320.6	3945.3	1403.1	752.5	822.2	2230.8	2061.4	126.8	302.8	4100.5	730.6	252.1	2137.6	3207.2
1985	4642.0	4223.8	1421.7	811.3	893.9	2418.9	2141.5	140.6	328.5	4290.4	784.6	271.2	2224.9	3449.2
1986	4976.4	4511.4	1440.5	874.4	973.0	2621.2	2224.5	155.6	356.6	4495.6	841.8	291.3	2317.1	3708.8
1987	5330.0	4819.1	1459.7	942.0	1059.3	2837.5	2311.8	171.8	387.2	4719.4	902.7	312.6	2413.9	3988.3
1988	5703.6	5150.1	1478.1	1013.2	1153.2	3067.6	2403.4	188.9	420.4	4955.9	967.2	335.4	2515.7	4289.3
1989	6095.9	5502.2	1495.1	1087.4	1255.0	3312.0	2499.0	206.7	456.2	5201.0	1035.2	359.3	2623.1	4612.8
1990	6506.5	5873.4	1510.8	1164.9	1365.1	3572.4	2598.6	224.9	494.7	5455.6	1106.7	384.4	2736.9	4960.1
1991	6937.5	6265.1	1525.6	1246.4	1483.8	3850.1	2702.7	243.6	535.9	5722.9	1182.1	410.8	2857.3	5332.9

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1980	572.2	74.9	157.7	0.0	620.9	54.1	N/A
1981	610.5	85.4	171.4	0.0	673.4	60.9	N/A
1982	657.0	95.9	186.4	0.0	719.3	67.1	N/A
1983	716.7	107.8	203.6	0.0	772.8	73.4	N/A
1984	783.6	121.2	221.2	0.0	835.4	80.5	N/A
1985	852.4	135.5	239.6	0.0	902.9	88.4	N/A
1986	923.0	150.4	259.0	0.0	972.3	96.9	N/A
1987	998.2	166.1	280.0	0.0	1044.6	105.8	N/A
1988	1079.5	182.8	302.6	0.0	1121.7	115.1	N/A
1989	1166.9	200.5	326.7	0.0	1204.9	125.1	N/A
1990	1259.7	219.3	352.3	0.0	1293.8	135.8	N/A
1991	1358.0	239.2	379.6	0.0	1388.4	147.2	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1980	9455.1	11154.7	67.216	1479.8
1981	9955.8	11859.9	68.999	1601.5
1982	10527.2	12719.1	72.852	1725.7
1983	11141.6	13655.2	79.626	1874.3
1984	11778.3	14615.2	87.389	2041.9
1985	12437.1	15605.5	94.846	2218.7
1986	13129.9	16658.3	102.183	2401.6
1987	13864.6	17790.7	110.152	2594.7
1988	14641.6	19000.3	119.094	2801.8
1989	15459.5	20281.3	128.880	3024.1
1990	16319.4	21635.9	139.318	3261.0
1991	17224.8	23071.8	150.406	3512.4

COPPER
SIMULATION # 2

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	2084.5	2013.6	1320.8	394.7	509.8	1176.0	1633.9	85.9	188.9	2797.0	465.1	119.6	1810.1	2253.2
1981	2092.7	1989.6	1350.5	405.4	517.0	1238.6	1665.0	83.1	190.5	2836.7	481.4	120.2	1866.4	2339.7
1982	2119.8	1995.2	1382.0	418.7	523.0	1296.7	1699.5	80.1	192.0	2895.0	499.1	121.9	1919.0	2431.5
1983	2153.3	2013.5	1410.5	428.7	528.6	1354.2	1734.3	76.2	193.4	2937.2	516.4	123.7	1970.4	2527.2
1984	2182.3	2023.9	1436.4	436.4	534.2	1411.6	1767.6	71.2	194.6	2963.5	533.1	125.0	2021.9	2625.2
1985	2209.5	2025.1	1462.2	444.1	540.0	1471.8	1800.0	65.5	195.5	2988.3	549.6	126.0	2073.2	2725.6
1986	2240.7	2024.9	1488.9	453.2	545.9	1534.8	1832.8	59.3	196.2	3020.2	566.4	127.1	2123.3	2829.3
1987	2278.2	2029.0	1515.9	463.1	551.7	1599.5	1866.2	52.8	197.0	3056.6	583.5	128.5	2171.8	2937.4
1988	2320.6	2036.3	1542.4	472.9	557.6	1665.5	1900.1	45.9	197.7	3092.3	600.9	129.9	2218.9	3050.1
1989	2366.2	2043.8	1568.3	482.4	563.8	1733.0	1934.1	38.4	198.4	3125.8	618.3	131.3	2264.9	3167.2
1990	2414.8	2050.4	1593.9	491.9	570.3	1802.4	1968.1	30.6	199.1	3158.5	635.8	132.6	2309.8	3288.9
1991	2467.3	2057.0	1619.4	501.7	577.3	1873.9	2002.3	22.4	199.9	3192.0	653.6	134.0	2353.3	3415.4

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1980	398.0	58.2	116.0	0.0	524.7	32.3	N/A
1981	391.7	60.7	120.4	0.0	534.2	33.8	N/A
1982	392.7	63.0	126.4	0.0	542.9	34.5	N/A
1983	399.4	65.6	132.2	0.0	556.0	35.2	N/A
1984	406.4	68.6	137.6	0.0	572.4	36.3	N/A
1985	411.3	71.5	142.9	0.0	588.1	37.5	N/A
1986	415.4	74.2	148.4	0.0	602.4	38.8	N/A
1987	420.4	76.9	154.3	0.0	616.5	40.0	N/A
1988	426.7	79.8	160.4	0.0	631.7	41.2	N/A
1989	433.5	82.7	166.7	0.0	648.0	42.5	N/A
1990	440.5	85.8	173.0	0.0	664.9	43.9	N/A
1991	447.5	88.9	179.6	0.0	682.2	45.4	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1980	8013.1	8840.0	50.804	1129.3
1981	8163.5	9013.3	49.797	1140.7
1982	8334.4	9239.1	50.940	1159.5
1983	8506.9	9460.7	53.358	1188.4
1984	8670.1	9656.9	55.596	1221.2
1985	8830.0	9846.4	57.188	1251.3
1986	8994.0	10048.8	58.608	1279.2
1987	9164.2	10266.8	60.316	1308.1
1988	9338.2	10492.8	62.347	1339.7
1989	9514.0	10722.0	64.506	1373.4
1990	9691.8	10955.5	66.675	1408.1
1991	9873.0	11196.5	68.883	1443.6

COPPER
SIMULATION # 3

	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
Year	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	2525.2	2479.5	1315.8	459.6	546.2	1194.6	1689.2	88.0	202.9	3071.7	496.6	150.6	1817.6	2329.3
1981	2626.8	2540.6	1342.0	487.5	569.5	1266.9	1733.2	89.5	210.3	3164.2	522.6	157.3	1881.6	2455.0
1982	2751.2	2635.1	1372.0	521.1	593.9	1335.5	1782.1	92.7	218.6	3290.9	551.4	165.6	1943.3	2592.2
1983	2888.6	2752.8	1399.2	553.0	620.1	1404.3	1833.2	96.4	227.8	3409.0	581.3	174.6	2004.7	2740.0
1984	3023.6	2869.4	1423.3	582.7	648.6	1472.4	1883.7	100.1	237.7	3511.0	611.4	183.3	2068.1	2896.3
1985	3156.6	2978.7	1446.3	612.6	679.7	1543.7	1933.8	103.7	248.2	3610.2	642.1	191.8	2133.4	3060.4
1986	3293.3	3087.9	1470.0	644.7	713.2	1618.3	1984.8	107.4	259.4	3718.7	673.9	200.6	2199.9	3233.5
1987	3437.7	3204.2	1494.1	678.7	749.1	1695.7	2037.4	111.4	271.3	3836.3	707.0	210.0	2267.0	3416.9
1988	3588.9	3328.3	1517.8	713.8	787.3	1775.1	2091.3	115.4	284.1	3957.7	741.2	219.7	2334.9	3611.2
1989	3745.3	3457.2	1540.6	749.5	828.1	1856.5	2146.2	119.1	297.5	4080.0	776.4	229.7	2404.1	3816.6
1990	3906.2	3589.3	1562.7	786.1	871.4	1940.4	2202.1	122.7	311.8	4204.5	812.6	240.0	2474.6	4033.6
1991	4072.7	3725.0	1584.5	824.0	917.4	2027.1	2259.1	125.9	326.8	4333.0	849.8	250.6	2546.4	4262.8

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1980	477.0	65.8	121.4	0.0	566.4	42.4	N/A
1981	490.1	71.8	126.8	0.0	594.5	46.0	N/A
1982	510.6	77.9	134.3	0.0	620.0	49.1	N/A
1983	539.1	84.7	141.8	0.0	650.4	52.1	N/A
1984	570.2	92.2	149.3	0.0	686.3	55.6	N/A
1985	600.5	100.0	156.5	0.0	723.4	59.4	N/A
1986	630.3	107.9	164.1	0.0	760.2	63.5	N/A
1987	661.8	116.0	172.1	0.0	797.6	67.6	N/A
1988	695.8	124.5	180.5	0.0	837.2	71.9	N/A
1989	731.6	133.4	189.2	0.0	879.2	76.4	N/A
1990	768.9	142.7	198.2	0.0	923.5	81.2	N/A
1991	807.4	152.4	207.4	0.0	969.5	86.1	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1980	8593.5	9773.2	57.452	1272.9
1981	8886.0	10160.9	57.510	1329.2
1982	9212.5	10633.1	59.689	1391.8
1983	9554.9	11130.0	63.718	1468.1
1984	9896.4	11615.1	68.043	1553.5
1985	10240.1	12101.1	71.830	1639.8
1986	10594.5	12611.2	75.415	1726.0
1987	10963.6	13153.2	79.357	1815.2
1988	11345.5	13721.1	83.814	1909.9
1989	11738.1	14308.7	88.617	2009.9
1990	12141.3	14916.5	93.610	2114.4
1991	12556.7	15548.4	98.801	2222.9

COPPER
SIMULATION #4

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	2469.6	2471.7	1316.3	458.9	541.9	1150.6	1682.3	88.5	201.3	3009.2	492.8	150.2	1816.8	2327.4
1981	2558.8	2532.1	1343.1	487.0	563.0	1212.2	1724.6	90.4	207.9	3086.0	517.5	156.9	1879.9	2452.5
1982	2670.4	2625.9	1373.4	520.5	585.0	1268.8	1771.8	94.1	215.4	3194.5	544.9	165.2	1940.2	2589.1
1983	2793.9	2741.9	1400.8	552.0	608.3	1325.0	1820.7	98.3	223.6	3291.8	573.1	174.0	2000.3	2736.1
1984	2914.6	2856.0	1425.0	581.3	633.8	1379.9	1869.0	102.4	232.3	3372.2	601.5	182.6	2062.1	2891.4
1985	3033.1	2962.6	1448.4	611.0	661.4	1437.2	1916.7	106.6	241.5	3449.3	630.3	191.0	2125.5	3054.4
1986	3155.4	3069.2	1472.5	642.9	691.2	1497.3	1965.3	111.1	251.4	3534.7	660.0	199.7	2189.8	3226.4
1987	3285.0	3183.0	1497.0	676.7	722.9	1559.2	2015.3	115.8	261.9	3627.8	691.0	208.9	2254.4	3408.6
1988	3421.2	3304.4	1521.1	711.5	756.8	1622.2	2066.5	120.5	273.0	3723.1	722.9	218.5	2319.5	3601.6
1989	3562.0	3430.2	1544.3	746.9	792.8	1686.4	2118.6	125.1	284.8	3817.9	755.7	228.4	2385.6	3805.6
1990	3707.1	3559.0	1566.9	783.2	831.1	1752.2	2171.5	129.6	297.2	3913.5	789.2	238.5	2452.7	4021.1
1991	3857.4	3691.3	1589.1	820.7	871.7	1819.9	2225.3	133.9	310.2	4011.7	823.8	248.9	2520.8	4248.7

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1980	473.9	65.5	117.1	0.0	559.9	42.1	N/A
1981	486.1	71.3	121.6	0.0	584.3	45.5	N/A
1982	506.0	77.2	128.1	0.0	606.3	48.5	N/A
1983	533.8	83.9	134.6	0.0	633.3	51.3	N/A
1984	563.9	91.2	140.8	0.0	665.3	54.7	N/A
1985	592.9	98.8	146.9	0.0	698.1	58.4	N/A
1986	621.5	106.6	153.1	0.0	730.3	62.3	N/A
1987	651.8	114.5	159.8	0.0	763.0	66.2	N/A
1988	684.4	122.8	166.7	0.0	797.4	70.3	N/A
1989	718.8	131.4	173.9	0.0	834.2	74.6	N/A
1990	754.5	140.5	181.3	0.0	872.7	79.1	N/A
1991	791.4	149.8	188.9	0.0	912.7	83.9	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1980	8521.0	9656.5	56.562	1258.4
1981	8794.9	10017.1	56.426	1308.8
1982	9101.0	10458.0	58.485	1366.2
1983	9420.8	10919.1	62.326	1436.8
1984	9738.2	11365.8	66.383	1515.9
1985	10056.9	11812.1	69.873	1595.1
1986	10385.5	12281.2	73.167	1673.8
1987	10727.4	12779.9	76.814	1755.2
1988	11080.9	13301.7	80.954	1841.6
1989	11443.6	13840.5	85.408	1932.9
1990	11815.6	14397.1	90.024	2028.1
1991	12198.3	14975.1	94.813	2126.7

COPPER
SIMULATION #5

Year	North America Supply	North America Demand	South America Supply	South America Demand	Asia Supply	Asia Demand	Africa Supply	Africa Demand	Europe Supply	Europe Demand	Oceania Supply	Oceania Demand	Centrally Planned Supply	Centrally Planned Demand
1980	2567.4	2529.6	1308.1	459.2	549.5	1362.3	1603.9	82.8	206.7	3044.4	496.9	152.3	1827.8	2342.1
1981	2672.1	2574.4	1332.5	485.9	574.7	1464.5	1645.5	82.0	215.2	3123.6	521.5	158.4	1899.9	2467.7
1982	2812.5	2661.2	1363.2	520.6	601.6	1573.3	1693.7	83.4	224.8	3254.0	550.3	166.7	1968.6	2604.9
1983	2971.8	2782.1	1391.8	554.3	630.4	1682.1	1745.2	85.8	235.4	3381.5	581.3	175.9	2036.3	2753.9
1984	3130.0	2906.2	1415.9	584.4	662.0	1792.4	1796.3	88.2	246.8	3485.9	612.8	184.9	2105.9	2911.8
1985	3283.7	3019.6	1438.2	614.0	696.8	1908.3	1846.6	90.5	259.0	3582.0	645.0	193.6	2177.9	3077.3
1986	3440.1	3129.7	1461.2	645.9	734.7	2031.3	1897.7	93.0	271.9	3687.2	678.6	202.5	2251.4	3251.4
1987	3605.1	3246.9	1485.0	680.3	775.4	2160.8	1950.6	95.7	285.7	3804.1	713.9	211.9	2325.5	3435.8
1988	3778.4	3373.7	1508.5	715.8	819.0	2295.8	2005.2	98.6	300.5	3926.0	750.8	221.8	2400.4	3631.3
1989	3957.9	3506.7	1531.0	752.0	865.7	2436.4	2061.1	101.2	316.1	4048.2	789.0	232.0	2476.7	3838.3
1990	4142.3	3642.9	1552.6	788.9	915.5	2583.5	2118.0	103.5	332.6	4171.3	828.3	242.5	2554.5	4056.9
1991	4332.7	3782.7	1573.8	827.1	968.5	2737.5	2176.2	105.5	350.0	4298.1	869.0	253.3	2634.0	4287.8

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1980	501.5	68.8	138.5	0.0	595.5	45.1	N/A
1981	513.1	75.3	146.7	0.0	626.7	49.4	N/A
1982	531.0	81.3	156.6	0.0	650.0	52.6	N/A
1983	559.7	88.1	167.5	0.0	679.3	55.7	N/A
1984	592.9	95.7	178.5	0.0	715.9	59.1	N/A
1985	625.0	103.8	189.5	0.0	755.1	63.1	N/A
1986	655.9	112.0	200.8	0.0	793.6	67.4	N/A
1987	688.2	120.3	213.0	0.0	832.4	71.7	N/A
1988	723.4	129.0	225.9	0.0	873.3	76.2	N/A
1989	761.0	138.1	239.4	0.0	917.2	80.9	N/A
1990	800.2	147.7	253.6	0.0	963.8	85.9	N/A
1991	840.8	157.7	268.4	0.0	1012.5	91.1	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1980	8560.3	9972.8	64.277	1349.3
1981	8861.5	10356.5	63.702	1411.0
1982	9214.7	10864.0	65.180	1471.5
1983	9592.0	11415.6	69.355	1550.3
1984	9969.6	11953.9	74.234	1642.2
1985	10347.1	12485.3	78.466	1736.5
1986	10735.5	13040.9	82.262	1829.7
1987	11141.2	13635.4	86.380	1925.6
1988	11562.9	14263.0	91.144	2027.7
1989	11997.3	14914.7	96.386	2136.7
1990	12443.8	15589.5	101.877	2251.2
1991	12904.1	16291.8	107.584	2370.5

COPPER
SIMULATION # 6

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	2736.2	2405.2	1321.6	453.9	563.4	1383.7	1689.4	93.4	204.2	3079.6	513.1	147.4	2172.1	2311.9
1981	2901.8	2444.0	1354.9	484.7	595.1	1493.7	1733.5	100.1	211.9	3205.6	549.7	153.6	2328.7	2430.2
1982	3096.0	2525.8	1390.9	519.6	629.4	1607.1	1783.5	109.4	220.6	3360.9	591.2	161.5	2483.0	2560.4
1983	3298.1	2629.4	1423.4	552.1	667.1	1720.5	1835.3	120.1	230.1	3502.1	635.3	169.9	2637.6	2701.1
1984	3494.2	2729.0	1452.2	581.9	708.6	1837.0	1886.5	131.4	240.4	3624.5	680.8	177.9	2794.4	2849.8
1985	3685.6	2820.1	1480.4	612.5	754.1	1959.7	1937.4	143.2	251.4	3746.2	728.2	185.8	2953.0	3006.1
1986	3879.6	2912.6	1509.4	645.6	803.2	2089.3	1989.6	155.7	263.2	3878.8	777.8	194.1	3112.3	3171.7
1987	4079.5	3014.2	1538.6	680.7	855.8	2224.7	2043.9	168.8	275.9	4020.4	829.6	202.9	3271.6	3347.8
1988	4284.2	3124.6	1567.1	716.7	911.8	2365.4	2099.8	182.0	289.5	4164.4	883.2	212.2	3431.0	3535.1
1989	4491.3	3240.2	1594.5	753.3	971.2	2511.7	2157.0	195.1	304.0	4308.2	938.2	221.8	3590.7	3733.8
1990	4700.7	3359.4	1621.1	790.9	1034.2	2664.5	2215.4	208.1	319.4	4453.6	994.6	231.6	3750.8	3944.3
1991	4913.7	3483.2	1647.1	829.7	1100.5	2824.1	2275.3	220.7	335.7	4602.6	1052.4	241.8	3911.2	4167.4

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1980	440.8	61.7	131.0	0.0	524.7	38.6	N/A
1981	438.5	65.5	138.2	0.0	531.4	40.1	N/A
1982	445.5	69.3	146.8	0.0	536.1	40.7	N/A
1983	461.3	74.0	156.0	0.0	547.9	41.4	N/A
1984	479.0	79.4	165.1	0.0	565.3	42.7	N/A
1985	495.4	85.0	174.3	0.0	583.8	44.4	N/A
1986	511.8	90.7	184.1	0.0	602.0	46.3	N/A
1987	530.5	96.7	194.8	0.0	621.5	48.3	N/A
1988	552.3	103.2	206.1	0.0	643.9	50.5	N/A
1989	576.4	110.2	218.1	0.0	669.5	53.0	N/A
1990	602.3	117.6	230.8	0.0	698.0	55.8	N/A
1991	630.1	125.4	244.1	0.0	729.0	59.0	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1980	9199.9	9875.1	46.522	1196.8
1981	9675.5	10311.8	42.338	1213.7
1982	10194.5	10844.7	40.764	1238.4
1983	10726.9	11395.2	41.173	1280.6
1984	11257.2	11931.6	41.659	1331.5
1985	11790.1	12473.7	41.529	1382.9
1986	12335.2	13047.8	41.320	1435.0
1987	12895.0	13659.6	41.668	1491.8
1988	13466.6	14300.4	42.689	1556.0
1989	14046.9	14964.2	44.168	1627.2
1990	14636.2	15652.3	45.960	1704.5
1991	15235.8	16369.5	48.104	1787.6

TABLE 10-3
SIMULATION EQUATIONS
PRIMARY PRODUCTION
ALUMINUM
(T-STATISTICS IN PARENTHESES)

		North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
θ_0	Constant	-559.148 (-1.2873)	-84.8237 (-2.2810)	-255.296 (-1.7846)	-82.5312 (-0.9231)			
θ_1	$P_{M,t}$	44.8745 (2.2249)	5.9285 (2.2910)					
θ_2	$P_{M,t-1}$	2.8725 (0.1634)	-3.1798 (-1.3588)	8.3518 (1.3579)	1.8953 (0.5965)			
θ_3	$ROWS_{M,t-1}$	-0.5353 (-4.3538)						
θ_4	$D_{M,t}$	0.3532 (9.6504)	0.0056 (2.0320)		0.0092 (2.3453)			
θ_5	$D_{M,t-1}$	0.3824 (4.4876)		0.0286 (1.9423)				
θ_6	t							
θ_7	r_t		NA	NA	NA			
θ_8	r_{t-1}		NA	NA	NA			
θ_9	$\sum^{t-2} Q^S_{M,i}$	-0.0042 (-0.2967)	0.0188 (0.7093)					
θ_{10}	$Q^S_{M,t-1}$	-0.0933 (-0.5025)	0.8048 (3.7668)	0.8414 (7.3740)	0.8017 (5.4352)			
	R^2	0.9957	0.9875	0.9885	0.9693			
	SEE	95.5451	12.7906	59.2330	19.2582			

10.4 ALUMINUM

The results of our in-sample predictions indicated that some of the equations would have to be modified. As was the case with copper, changes were required in primary production, demand, and the price equation.

In the case of primary production, alternative equations were estimated for North America, South America, Asia, and Africa. These equations are presented in Table 10-3. In them, variables that proved to be inappropriate in the simulation model were eliminated.[4]

For demand, only the equation for North America was reestimated. The results of this estimation are as follows:

$$\begin{aligned} Q^{D}_{M,t} = {} & \underset{(-8.5909)}{-5287.82} \; \underset{(-1.4754)}{-\;7.4294}(P_{M,t}-P_{R,t}) + \underset{(2.2405)}{11.5189}\,(P_{M,t-1}-P_{R,t-1}) \\ & + \underset{(9.0624)}{9234.44}Y_{t} + \underset{(0.4614)}{0.0537}\,Q^{D}_{M,t-1} \; \underset{(-3.7794)}{-\;783.560}\text{ Dummy} \end{aligned}$$

$$R^{2} = 0.9827 \qquad \text{SEE } 200.530$$

In this, some of the explanatory variables were eliminated and a dummy variable was added to account for the recessionary effects of 1975-1976.

The alternative price equation was estimated as follows:

$$\begin{aligned} P_{M,t} = {} & \underset{(3.3400)}{16.8802} + \underset{(4.6839)}{1.0090}\,P_{M,t-1} \; \underset{(-1.8692)}{-\;0.3765}P_{M,t-2} \\ & + \underset{(0.0697)}{0.00007}(S_{M,t-1} + WQ^{SCRAP}_{M,t-1} - D_{M,t-1}) \\ & + \underset{(2.5799)}{1.1989}r_{t} + \underset{(1.6286)}{0.0458}P_{R,t} \; \underset{(-2.1385)}{-\;17.8384}Y_{t} \end{aligned}$$

$$R^{2} = 0.7959 \qquad \text{SEE} = 1.1252$$

As with copper, we changed the inventory variable to include scrap. Furthermore, several of the explanatory variables reported in Table 7-8 proved unsatisfactory in the context of the simulation model and were eliminated.

Using our modified model, we simulated production, consumption, and price over the period 1965-1977. These in-sample predictions are presented as Simulation #00. The values presented in this simulation can be compared with those in Tables 7-1 through 7-4.

Finally, the simulations of future values are presented as Simulation #0 through Simulation #6. The growth rates for the country specific variables are as reported in section 10.2. Over the same period, the mean rate of change in the price of copper was +0.044. As we have noted earlier, we leave the selection of the most appropriate simulation to the reader.

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[4] These equations can be compared with those presented in Table 7-5.

ALUMINUM
SIMULATION # 00

	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
Year	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1965	3261.3	2852.4	60.7	127.9	410.1	566.4	66.8	40.5	1274.7	1624.8	88.1	84.8	1458.2	1277.6
1966	3478.9	3084.0	76.3	152.2	488.9	686.1	85.9	48.1	1414.4	1820.6	103.7	91.5	1561.8	1394.5
1967	3727.7	3320.8	92.0	178.6	573.1	787.8	106.7	54.5	1567.2	1986.3	122.5	101.4	1699.3	1507.9
1968	3927.1	3562.0	108.9	201.4	659.3	890.2	128.7	61.6	1730.4	2142.1	144.4	110.6	1837.9	1627.5
1969	4128.2	3809.6	127.7	225.4	747.5	996.7	151.8	68.7	1900.2	2292.2	169.2	120.4	1975.2	1752.2
1970	4336.7	4064.5	148.6	250.1	838.8	1108.1	176.1	75.9	2077.9	2441.1	197.1	130.3	2117.8	1882.5
1971	4549.6	4327.1	171.8	275.3	933.9	1224.7	201.9	83.3	2265.2	2591.8	228.1	140.6	2265.4	2019.5
1972	4767.3	4597.6	197.3	300.9	1033.5	1346.8	229.1	90.9	2463.1	2746.2	262.2	151.1	2417.8	2163.8
1973	4989.7	4875.7	225.4	327.2	1138.0	1474.6	257.9	98.7	2672.9	2905.3	299.2	162.0	2575.3	2315.9
1974	5216.8	5161.7	256.2	354.0	1247.5	1608.5	288.2	106.6	2895.6	3069.8	338.9	173.1	2738.2	2476.6
1975	5172.4	4672.0	285.6	381.4	1362.3	1748.6	312.8	114.7	3086.6	3240.1	366.6	184.6	2838.8	2646.1
1976	5206.1	4931.8	319.1	397.8	1460.1	1895.3	340.3	124.8	3277.1	3435.3	401.0	196.6	2984.8	2840.7
1977	5531.9	5240.2	356.7	435.4	1567.0	2048.9	370.6	132.0	3511.0	3612.8	438.3	208.5	3133.4	3025.6

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1965	699.1	11.4	161.9	3.6	560.7	10.8	N/A
1966	746.1	16.4	210.4	4.2	614.3	11.8	N/A
1967	795.3	20.2	238.0	4.8	662.3	14.3	N/A
1968	850.3	23.5	265.9	5.3	709.6	17.0	N/A
1969	910.4	26.7	295.5	5.8	756.0	18.9	N/A
1970	974.7	30.1	326.9	6.4	802.5	21.0	N/A
1971	1042.6	33.5	360.1	6.9	849.9	23.0	N/A
1972	1113.7	37.0	395.0	7.5	898.5	25.0	N/A
1973	1187.8	40.5	431.7	8.1	948.8	27.1	N/A
1974	1264.7	44.1	470.4	8.7	1000.8	29.3	N/A
1975	1262.1	47.7	510.9	9.3	1054.6	31.5	N/A
1976	1339.1	50.0	553.4	10.0	1115.3	33.9	N/A
1977	1378.5	54.1	598.0	10.6	1172.2	36.2	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1965	6619.9	6574.4	25.284	1447.4
1966	7209.8	7277.0	25.010	1603.2
1967	7888.4	7937.3	24.589	1734.8
1968	8536.6	8595.5	24.220	1871.6
1969	9199.8	9265.2	23.962	2013.4
1970	9893.0	9952.5	23.806	2161.5
1971	10615.7	10662.3	23.719	2315.9
1972	11370.3	11397.3	23.673	2476.7
1973	12158.4	12159.5	23.652	2644.0
1974	12981.5	12950.3	23.653	2817.9
1975	13425.0	12987.7	23.677	2916.2
1976	13988.6	13822.2	23.757	3101.6
1977	14908.7	14703.4	23.838	3249.6

ALUMINUM
SIMULATION # 0

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	5909.1	5940.9	463.1	587.6	1809.2	2588.5	447.7	146.8	4218.9	3836.4	518.7	219.3	3536.2	3323.1
1981	6185.2	6277.3	511.1	622.4	1922.6	2774.1	479.2	155.4	4531.7	4010.6	559.8	232.1	3730.3	3539.3
1982	6473.8	6625.6	565.1	657.8	2045.1	2969.0	513.6	164.3	4864.4	4198.0	606.4	245.2	3932.3	3768.9
1983	6773.5	6985.5	625.2	693.9	2178.4	3173.0	551.3	173.5	5217.2	4396.9	659.0	258.6	4145.1	4011.8
1984	7077.6	7356.3	691.4	730.7	2323.1	3386.7	592.2	183.0	5591.5	4606.7	717.1	272.5	4367.5	4269.2
1985	7385.3	7737.7	763.9	768.1	2479.1	3610.4	636.2	192.7	5988.6	4826.5	780.6	286.7	4599.1	4541.8
1986	7697.1	8129.8	843.1	806.2	2646.2	3844.6	683.3	202.6	6409.7	5056.1	849.1	301.3	4840.1	4830.3
1987	8013.7	8532.8	929.5	845.1	2824.4	4089.8	733.2	212.8	6855.8	5295.1	922.2	316.4	5090.9	5135.6
1988	8335.8	8947.1	1023.7	884.7	3013.7	4346.6	786.0	223.3	7328.3	5543.6	999.9	331.8	5352.0	5458.6
1989	8664.1	9372.9	1126.3	925.0	3214.2	4615.4	841.8	234.0	7828.3	5801.8	1082.2	347.7	5624.0	5800.2
1990	8999.1	9810.7	1237.9	966.1	3426.3	4896.8	900.6	245.0	8357.2	6069.9	1169.0	364.1	5907.3	6161.4
1991	9340.9	10260.9	1359.4	1008.0	3650.4	5191.5	962.4	256.3	8916.6	6348.4	1260.6	380.9	6202.6	6543.4

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1980	1491.2	75.1	751.4	12.0	1243.8	39.1	N/A
1981	1570.0	80.5	805.8	12.5	1296.6	41.0	N/A
1982	1656.1	85.3	862.4	13.1	1353.8	43.4	N/A
1983	1748.3	90.2	921.7	13.7	1415.3	45.9	N/A
1984	1845.9	95.0	983.8	14.4	1481.0	48.6	N/A
1985	1948.1	99.8	1049.0	15.1	1550.5	51.4	N/A
1986	2054.6	104.8	1117.2	15.9	1623.6	54.2	N/A
1987	2165.2	109.8	1188.7	16.7	1700.1	57.1	N/A
1988	2279.7	114.9	1263.7	17.5	1780.0	60.2	N/A
1989	2398.1	120.1	1342.2	18.4	1863.3	63.3	N/A
1990	2520.4	125.4	1424.4	19.3	1950.2	66.5	N/A
1991	2646.6	130.7	1510.5	20.3	2040.7	69.8	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1980	16902.9	16642.4	21.513	3612.5
1981	17919.9	17611.3	21.434	3806.5
1982	19000.8	18628.7	21.572	4014.2
1983	20149.6	19693.3	21.820	4235.2
1984	21360.4	20804.9	22.118	4468.6
1985	22632.9	21963.9	22.446	4713.8
1986	23968.5	23170.9	22.807	4970.2
1987	25369.7	24427.6	23.214	5237.6
1988	26839.4	25735.6	23.678	5515.9
1989	28380.8	27097.1	24.212	5805.4
1990	29997.4	28514.2	24.820	6106.1
1991	31692.8	29989.4	25.509	6418.6

ALUMINUM
SIMULATION # 1

Year	North America Supply	North America Demand	South America Supply	South America Demand	Asia Supply	Asia Demand	Africa Supply	Africa Demand	Europe Supply	Europe Demand	Oceania Supply	Oceania Demand	Centrally Planned Supply	Centrally Planned Demand
1980	6977.8	7277.6	454.0	671.7	1865.2	3134.7	487.2	179.6	4588.2	4266.5	567.2	256.4	3790.5	3482.7
1981	7527.3	8015.6	495.9	737.5	2001.2	3425.1	531.9	200.2	5046.4	4560.7	620.2	280.9	4057.5	3765.7
1982	8115.4	8796.3	543.2	806.2	2146.8	3734.6	580.6	221.6	5540.8	4877.1	678.4	306.6	4340.6	4071.4
1983	8739.9	9621.8	595.7	878.1	2304.7	4064.5	633.7	244.0	6073.4	5214.8	742.6	333.6	4642.4	4400.5
1984	9395.4	10493.9	653.6	953.1	2475.9	4416.3	691.3	267.5	6646.9	5573.8	813.1	362.0	4962.1	4754.6
1985	10081.9	11414.4	716.9	1031.5	2661.1	4791.2	753.6	292.0	7264.3	5954.3	889.7	391.8	5299.7	5135.2
1986	10800.5	12385.7	785.9	1113.4	2860.5	5190.9	820.5	317.5	7928.7	6356.5	972.3	423.1	5656.0	5544.3
1987	11553.4	13410.4	861.0	1199.1	3074.2	5616.9	892.2	344.1	8643.2	6781.0	1060.8	455.9	6031.9	5983.6
1988	12343.2	14491.3	942.7	1288.8	3302.5	6071.1	968.7	371.9	9411.1	7228.8	1155.3	490.4	6428.7	6455.5
1989	13172.5	15631.6	1031.4	1382.5	3546.1	6555.3	1050.2	400.9	10235.8	7700.7	1255.9	526.6	6847.7	6962.0
1990	14043.9	16834.7	1127.9	1480.6	3805.5	7071.4	1136.9	431.1	11121.1	8197.8	1362.7	564.6	7290.2	7505.9
1991	14960.1	18104.0	1232.7	1583.2	4081.5	7621.5	1229.0	462.6	12071.0	8721.6	1476.0	604.6	7757.7	8089.6

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1980	1660.8	88.5	914.7	12.5	1355.1	43.1	N/A
1981	1814.3	98.7	997.2	13.2	1437.6	47.0	N/A
1982	1986.2	108.8	1085.0	14.0	1526.1	51.5	N/A
1983	2174.6	119.3	1178.6	14.8	1621.0	56.4	N/A
1984	2378.2	130.2	1278.5	15.7	1722.6	61.6	N/A
1985	2596.6	141.4	1385.1	16.6	1830.8	67.0	N/A
1986	2829.2	153.2	1498.9	17.6	1945.4	72.8	N/A
1987	3076.3	165.6	1620.3	18.6	2066.7	78.9	N/A
1988	3338.0	178.5	1749.7	19.6	2194.6	85.3	N/A
1989	3614.8	192.1	1887.7	20.7	2329.5	92.0	N/A
1990	3907.4	206.2	2034.9	21.8	2471.6	99.0	N/A
1991	4216.4	221.1	2191.7	23.0	2621.3	106.4	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1980	18730.1	19269.2	16.282	4074.7
1981	20280.5	20985.7	14.149	4408.0
1982	21946.0	22813.9	12.110	4771.5
1983	23732.3	24757.5	10.059	5164.6
1984	25638.3	26821.1	7.912	5586.7
1985	27667.1	29010.4	5.627	6037.5
1986	29824.3	31331.3	3.183	6517.2
1987	32116.7	33791.1	0.577	7026.3
1988	34552.2	36397.7	−2.193	7565.7
1989	37139.6	39159.5	−5.130	8136.8
1990	39888.2	42086.0	−8.240	8740.9
1991	42807.9	45187.0	−11.533	9379.9

ALUMINUM
SIMULATION #2

Year	North America Supply	North America Demand	South America Supply	South America Demand	Asia Supply	Asia Demand	Africa Supply	Africa Demand	Europe Supply	Europe Demand	Oceania Supply	Oceania Demand	Centrally Planned Supply	Centrally Planned Demand
1980	4908.7	4704.9	471.7	506.8	1755.2	2071.7	410.4	115.8	3867.2	3435.6	472.9	184.3	3296.1	3169.7
1981	4959.2	4712.0	524.8	514.0	1848.1	2170.2	430.3	114.2	4051.7	3509.0	503.8	187.0	3428.3	3324.8
1982	5009.3	4722.0	584.5	520.5	1950.3	2272.2	452.8	112.8	4246.5	3591.7	541.1	189.6	3563.3	3486.0
1983	5059.4	4733.6	650.5	526.9	2062.7	2377.3	477.9	111.5	4451.2	3681.4	584.3	192.2	3705.0	3653.6
1984	5102.9	4745.9	722.9	532.6	2185.2	2485.3	505.7	110.4	4666.6	3777.0	633.0	194.9	3852.1	3828.4
1985	5140.0	4758.2	801.8	538.0	2317.2	2596.4	535.7	109.4	4893.3	3877.4	686.4	197.5	4003.9	4010.7
1986	5171.3	4770.4	887.8	542.8	2458.2	2710.6	567.8	108.4	5131.6	3982.0	743.9	200.2	4160.8	4200.9
1987	5197.7	4782.5	981.3	547.2	2607.9	2827.9	601.9	107.6	5381.9	4090.3	805.4	202.9	4322.9	4399.2
1988	5219.7	4794.5	1083.0	551.0	2766.2	2948.6	637.9	107.0	5644.4	4202.1	870.4	205.7	4490.6	4606.1
1989	5237.7	4806.6	1193.4	554.4	2933.1	3072.7	675.7	106.4	5919.7	4317.3	939.0	208.4	4664.2	4821.7
1990	5251.7	4818.8	1313.4	557.1	3108.7	3200.2	715.5	106.0	6208.2	4436.1	1011.1	211.2	4843.8	5046.5
1991	5261.5	4831.2	1443.5	559.3	3293.3	3331.2	757.3	105.7	6510.4	4558.4	1086.7	214.0	5029.7	5280.7

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1980	1332.2	62.2	596.4	11.5	1140.0	35.2	N/A
1981	1345.4	63.3	627.8	11.9	1168.1	35.5	N/A
1982	1358.9	63.5	659.7	12.3	1200.0	36.0	N/A
1983	1372.9	63.7	692.4	12.8	1235.3	36.5	N/A
1984	1387.3	63.7	726.0	13.3	1273.6	37.1	N/A
1985	1402.3	63.5	760.5	13.9	1314.3	37.7	N/A
1986	1417.8	63.3	796.0	14.5	1357.0	38.4	N/A
1987	1433.8	63.1	832.5	15.1	1401.6	39.0	N/A
1988	1450.5	62.7	870.1	15.8	1448.0	39.6	N/A
1989	1467.8	62.2	908.7	16.6	1496.1	40.3	N/A
1990	1485.8	61.6	948.5	17.4	1545.9	40.9	N/A
1991	1504.6	60.8	989.4	18.2	1597.6	41.6	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1980	15182.2	14188.9	26.441	3177.5
1981	15746.2	14531.2	28.134	3252.1
1982	16347.7	14894.8	30.054	3330.6
1983	16991.0	15276.5	32.090	3413.6
1984	17668.3	15674.4	34.195	3501.0
1985	18378.2	16087.5	36.366	3592.2
1986	19121.3	16515.3	38.619	3687.0
1987	19898.8	16957.7	40.973	3785.1
1988	20712.1	17414.9	43.443	3886.7
1989	21562.8	17887.5	46.042	3991.6
1990	22452.3	18375.8	48.774	4100.1
1991	23382.4	18880.5	51.648	4212.3

ALUMINUM
SIMULATION #3

Year	North America Supply	North America Demand	South America Supply	South America Demand	Asia Supply	Asia Demand	Africa Supply	Africa Demand	Europe Supply	Europe Demand	Oceania Supply	Oceania Demand	Centrally Planned Supply	Centrally Planned Demand
1980	5703.0	5940.9	457.5	585.7	1789.2	2146.5	438.7	147.1	4152.5	3842.5	500.9	219.3	3475.3	3328.1
1981	5960.1	6277.4	503.6	620.7	1893.4	2266.0	467.4	155.7	4449.9	4016.6	536.4	232.1	3660.1	3544.3
1982	6224.6	6625.6	555.6	656.0	2006.2	2390.5	498.9	164.6	4767.3	4204.2	577.4	245.2	3851.2	3774.0
1983	6498.5	6985.4	613.4	692.2	2129.2	2519.2	533.5	173.8	5103.7	4403.2	624.2	258.7	4052.3	4017.0
1984	6775.0	7356.2	677.2	728.9	2263.0	2652.2	571.3	183.3	5460.1	4613.0	676.6	272.5	4262.3	4274.5
1985	7053.2	7737.6	747.0	766.3	2407.5	2789.6	611.9	193.0	5837.7	4832.9	734.1	286.7	4480.5	4547.1
1986	7333.3	8129.7	823.3	804.4	2562.5	2931.6	655.4	202.9	6237.5	5062.5	796.4	301.3	4707.3	4835.7
1987	7616.2	8532.7	906.5	843.3	2727.7	3078.2	701.5	213.1	6660.5	5301.5	863.2	316.4	4942.8	5141.0
1988	7902.4	8946.9	997.0	882.9	2903.2	3229.7	750.4	223.6	7107.7	5550.1	934.4	331.9	5187.7	5464.1
1989	8192.7	9372.7	1095.6	923.2	3089.1	3386.1	801.9	234.3	7580.4	5808.4	1009.8	347.8	5442.2	5805.8
1990	8487.2	9810.5	1202.8	964.3	3285.7	3547.7	856.1	245.3	8079.6	6076.6	1089.6	364.1	5706.9	6167.1
1991	8786.3	10260.7	1319.4	1006.2	3493.1	3714.7	913.1	256.6	8606.8	6355.0	1173.7	380.9	5982.2	6549.0

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1980	1491.2	74.7	617.2	12.0	1245.5	39.1	N/A
1981	1570.0	80.3	654.0	12.5	1298.4	41.0	N/A
1982	1656.0	85.1	691.1	13.1	1355.7	43.4	N/A
1983	1748.2	89.9	729.0	13.8	1417.2	45.9	N/A
1984	1845.7	94.8	768.0	14.4	1482.9	48.6	N/A
1985	1947.9	99.6	808.2	15.1	1552.4	51.4	N/A
1986	2054.4	104.6	849.8	15.9	1625.4	54.2	N/A
1987	2164.9	109.6	892.6	16.7	1702.0	57.1	N/A
1988	2279.4	114.7	937.0	17.5	1781.9	60.2	N/A
1989	2397.7	119.9	982.8	18.4	1865.2	63.3	N/A
1990	2520.0	125.1	1030.1	19.4	1952.0	66.5	N/A
1991	2646.2	130.5	1079.1	20.3	2042.5	69.8	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1980	16517.2	16210.2	21.509	3479.7
1981	17470.9	17112.9	21.420	3656.3
1982	18481.1	18060.1	21.549	3844.4
1983	19554.8	19049.4	21.790	4044.0
1984	20685.4	20080.5	22.083	4254.4
1985	21872.0	21153.2	22.405	4474.6
1986	23115.6	22268.1	22.760	4704.2
1987	24418.4	23426.3	23.160	4942.9
1988	25782.8	24629.1	23.618	5190.6
1989	27211.7	25878.3	24.143	5447.2
1990	28708.0	27175.6	24.743	5713.1
1991	30274.8	28523.1	25.422	5988.3

ALUMINUM
SIMULATION # 4

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	5594.9	5941.0	454.7	584.7	1779.3	2031.1	434.1	147.2	4118.9	3725.5	491.8	219.3	3444.4	3330.6
1981	5831.5	6277.4	499.6	619.6	1878.4	2129.3	461.1	155.9	4406.2	3869.0	524.0	232.2	3621.5	3547.2
1982	6074.7	6625.7	550.3	654.8	1985.4	2231.4	490.8	164.8	4712.7	4024.4	561.3	245.2	3804.3	3777.1
1983	6326.8	6985.4	606.8	690.9	2102.0	2336.4	523.4	174.0	5037.2	4189.3	604.3	258.7	3996.6	4020.4
1984	6580.6	7356.1	668.9	727.6	2228.7	2444.4	558.9	183.5	5380.8	4363.1	652.5	272.5	4197.3	4278.0
1985	6835.4	7737.6	737.0	765.0	2365.5	2555.5	597.2	193.2	5744.5	4545.0	705.8	286.7	4405.7	4550.8
1986	7091.2	8129.6	811.2	803.1	2512.1	2669.7	638.1	203.1	6129.2	4734.5	763.6	301.4	4622.0	4839.6
1987	7348.9	8532.6	892.2	841.9	2668.3	2787.1	681.6	213.3	6535.9	4931.3	825.7	316.4	4846.5	5145.1
1988	7608.9	8946.8	980.3	881.4	2834.1	2907.7	727.6	223.8	6965.6	5135.3	892.0	331.9	5079.6	5468.3
1989	7871.9	9372.6	1076.2	921.8	3009.7	3031.8	776.0	234.5	7419.3	5346.6	962.4	347.8	5321.7	5810.0
1990	8138.1	9810.4	1180.5	962.9	3195.2	3159.3	827.1	245.5	7898.1	5565.3	1036.9	364.1	5573.4	6171.4
1991	8407.8	10260.5	1293.8	1004.8	3390.9	3290.4	880.7	256.8	8403.3	5791.7	1115.7	381.0	5834.9	6553.3

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1980	1491.2	74.6	582.6	12.0	1212.8	39.1	N/A
1981	1569.9	80.2	613.3	12.5	1256.4	41.0	N/A
1982	1656.0	85.0	644.0	13.1	1303.7	43.4	N/A
1983	1748.2	89.8	675.1	13.8	1354.6	45.9	N/A
1984	1845.6	94.6	706.9	14.4	1409.0	48.6	N/A
1985	1947.8	99.4	739.5	15.1	1466.7	51.4	N/A
1986	2054.3	104.4	772.9	15.9	1527.2	54.2	N/A
1987	2164.7	109.4	807.3	16.7	1590.4	57.1	N/A
1988	2279.2	114.5	842.7	17.5	1656.4	60.2	N/A
1989	2397.5	119.7	879.1	18.4	1724.9	63.3	N/A
1990	2519.7	125.0	916.5	19.4	1796.3	66.5	N/A
1991	2645.9	130.3	955.0	20.3	1870.4	69.8	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1980	16318.0	15979.4	21.507	3412.3
1981	17222.2	16830.5	21.414	3573.5
1982	18179.5	17723.3	21.539	3745.2
1983	19196.9	18655.1	21.776	3927.3
1984	20267.8	19625.3	22.064	4119.2
1985	21390.9	20633.8	22.382	4319.9
1986	22567.4	21681.0	22.732	4528.8
1987	23799.0	22767.7	23.127	4745.7
1988	25088.0	23895.3	23.579	4970.4
1989	26437.2	25065.1	24.098	5202.9
1990	27849.3	26278.9	24.692	5443.2
1991	29327.1	27538.4	25.364	5691.6

ALUMINUM
SIMULATION # 5

	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
Year	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	5932.1	5940.9	463.0	588.6	1809.1	2588.5	389.9	146.6	4216.9	3833.7	518.1	219.3	3532.5	3320.8
1981	6213.5	6277.3	510.9	623.7	1922.3	2774.1	411.2	155.2	4529.2	4007.1	559.0	232.1	3725.9	3536.5
1982	6507.0	6625.6	564.9	659.2	2044.6	2969.0	436.1	164.1	4861.3	4193.8	605.3	245.2	3927.1	3765.4
1983	6811.2	6985.4	625.0	695.6	2177.7	3173.0	464.6	173.3	5213.5	4392.1	657.6	258.6	4139.3	4007.8
1984	7119.7	7356.2	691.1	732.5	2322.2	3386.7	496.5	182.7	5587.3	4601.1	715.5	272.4	4361.0	4264.6
1985	7431.8	7737.7	763.5	770.1	2478.0	3610.4	531.4	192.4	5983.8	4820.3	778.8	286.7	4591.9	4536.6
1986	7748.0	8129.8	842.7	808.5	2644.9	3844.6	569.1	202.3	6404.2	5049.2	846.9	301.3	4832.2	4824.6
1987	8069.1	8532.8	929.0	847.5	2822.8	4089.8	609.5	212.5	6849.7	5287.5	919.8	316.3	5082.3	5129.3
1988	8395.8	8947.0	1023.1	887.3	3011.9	4346.6	652.4	222.9	7321.5	5535.4	997.3	331.8	5342.7	5451.7
1989	8729.0	9372.8	1125.6	927.9	3212.1	4615.4	697.7	233.6	7820.7	5792.8	1079.3	347.7	5613.9	5792.8
1990	9068.9	9810.7	1237.2	969.2	3424.0	4896.8	745.6	244.6	8348.9	6060.2	1166.0	364.1	5896.4	6153.4
1991	9416.1	10260.8	1358.6	1011.3	3647.8	5191.5	796.1	255.8	8907.4	6337.9	1257.3	380.9	6190.8	6534.6

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1980	1491.2	75.3	751.4	12.0	1243.0	39.1	N/A
1981	1570.0	80.7	805.8	12.5	1295.7	41.0	N/A
1982	1656.0	85.6	862.4	13.1	1352.6	43.4	N/A
1983	1748.3	90.4	921.7	13.7	1413.9	45.9	N/A
1984	1845.8	95.2	983.8	14.4	1479.3	48.6	N/A
1985	1948.0	100.1	1048.9	15.1	1548.6	51.3	N/A
1986	2054.5	105.1	1117.2	15.9	1621.4	54.2	N/A
1987	2165.1	110.1	1188.7	16.7	1697.7	57.1	N/A
1988	2279.6	115.3	1263.7	17.5	1777.4	60.2	N/A
1989	2398.0	120.5	1342.2	18.4	1860.5	63.3	N/A
1990	2520.3	125.8	1424.4	19.3	1947.1	66.5	N/A
1991	2646.5	131.2	1510.5	20.3	2037.4	69.8	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1980	16861.7	16638.4	21.506	3611.9
1981	17871.9	17606.0	21.425	3805.7
1982	18946.4	18622.3	21.561	4013.1
1983	20088.8	19685.8	21.808	4233.9
1984	21293.2	20796.3	22.104	4467.1
1985	22559.2	21954.2	22.431	4712.1
1986	23888.1	23160.2	22.790	4968.3
1987	25282.2	24415.8	23.195	5235.4
1988	26744.6	25722.7	23.658	5513.5
1989	28278.4	27083.0	24.190	5802.7
1990	29887.0	28498.9	24.797	6103.3
1991	31574.0	29972.8	25.484	6415.6

ALUMINUM
SIMULATION # 6

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	5848.9	5940.9	463.5	585.1	1810.1	2588.5	448.0	147.2	4224.5	3844.1	520.8	219.4	3681.0	3329.5
1981	6119.5	6277.4	511.5	619.5	1923.9	2774.1	479.6	155.9	4538.2	4019.2	562.3	232.1	3883.0	3546.6
1982	6404.7	6625.7	565.6	654.8	2046.7	2969.0	514.2	164.8	4871.5	4207.4	609.3	245.2	4093.3	3776.6
1983	6700.4	6985.6	625.8	690.8	2180.4	3173.0	551.9	174.0	5225.0	4407.0	662.2	258.7	4315.1	4020.2
1984	7000.3	7356.4	692.1	727.3	2325.4	3386.7	592.9	183.5	5600.1	4617.4	720.6	272.5	4546.9	4278.1
1985	7303.9	7737.8	764.7	764.6	2481.8	3610.4	637.0	193.2	5998.0	4837.9	784.4	286.7	4788.2	4551.2
1986	7611.3	8129.9	844.0	802.5	2649.2	3844.6	684.1	203.2	6419.9	5068.1	853.0	301.4	5039.2	4840.3
1987	7923.2	8533.0	930.5	841.1	2827.7	4089.8	734.1	213.5	6866.9	5307.9	926.4	316.4	5300.6	5146.2
1988	8240.6	8947.2	1024.8	880.5	3017.3	4346.6	787.0	223.9	7340.3	5557.1	1004.3	331.9	5572.6	5469.9
1989	8564.0	9373.0	1127.5	920.6	3218.1	4615.4	842.9	234.7	7841.2	5816.1	1086.7	347.8	5856.0	5812.1
1990	8893.8	9810.9	1239.3	961.4	3430.5	4896.8	901.7	245.8	8371.2	6085.0	1173.8	364.2	6151.2	6174.0
1991	9230.4	10261.0	1360.8	1003.0	3654.9	5191.5	963.6	257.1	8931.6	6364.3	1265.6	381.0	6458.8	6556.6

SECONDARY PRODUCTION

Year	North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
1980	1491.3	74.8	751.4	12.0	1246.0	39.1	N/A
1981	1570.1	80.2	805.8	12.5	1299.3	41.0	N/A
1982	1656.2	84.9	862.4	13.1	1356.7	43.4	N/A
1983	1748.5	89.7	921.7	13.8	1418.5	45.9	N/A
1984	1846.0	94.5	983.8	14.4	1484.4	48.6	N/A
1985	1948.3	99.4	1049.0	15.2	1554.1	51.4	N/A
1986	2054.8	104.3	1117.2	15.9	1627.4	54.2	N/A
1987	2165.4	109.3	1188.8	16.7	1704.2	57.2	N/A
1988	2279.9	114.3	1263.7	17.6	1784.3	60.2	N/A
1989	2398.3	119.5	1342.2	18.5	1867.9	63.3	N/A
1990	2520.6	124.7	1424.4	19.4	1955.1	66.5	N/A
1991	2646.9	130.0	1510.6	20.4	2045.8	69.8	N/A

WORLD

Year	Production	Consumption	Price	Scrap
1980	16996.7	16654.7	21.536	3614.6
1981	18018.0	17624.9	21.457	3808.9
1982	19105.3	18643.5	21.596	4016.8
1983	20260.8	19709.2	21.845	4238.1
1984	21478.4	20821.8	22.144	4471.8
1985	22757.8	21981.8	22.473	4717.2
1986	24100.7	23190.0	22.836	4973.8
1987	25509.4	24447.8	23.244	5241.4
1988	26986.9	25757.1	23.711	5520.0
1989	28536.4	27119.7	24.246	5809.7
1990	30161.5	28538.1	24.856	6110.7
1991	31865.8	30014.6	25.547	6423.5

10.5 NICKEL

As a result of our in-sample predictions, it was necessary to alter several of our estimated equations. Changes were necessary for primary production, demand, and the price equation.

In the case of primary production, alternate equations were estimated for North America, South America, Africa, Oceania, and the Centrally Planned Economies. These equations are presented in Table 10-4. For North America, the current interest rate was eliminated. For South America, two dummy variables were included. The first of these reflects the fact that, after 1962, production from Cuba was included in the figures for the Centrally Planned Economies.[5] The second reflects the increased production in South America in 1972 and beyond.[6] For Africa, we excluded current price and added a dummy variable to reflect the increase in production that began in 1970.[7] For Oceania, a dummy variable was added to reflect the increased production beginning in 1968.[8] Finally, for the Centrally Planned Economies, a dummy variable was again added to reflect the fact that, after 1962, Cuban production was reported as part of the Centrally Planned Economies.[9]

In the case of demand, alternate equations were estimated for North America, Africa, Europe, Oceania, and the Centrally Planned Economies. These equations are presented in Table 10-5. For North America, we added a dummy variable to reflect the recessionary effects of 1975-1976.[10] For Africa, we reformulated our equation, eliminating the inventory variable and the rate of growth of the capital stock. For Europe, we eliminated current prices and added the dummy variable for the recession of 1975-1976. In Oceania, we added lagged prices and our inventory variable and again included the dummy variable to reflect the recessionary effects of 1975-1976. Finally, for the Centrally Planned Economies, we added a dummy variable to reflect the increase in consumption in 1963 and beyond.[11]

Our alternate price equation was estimated as follows:

$$P_{M,t} = \underset{(2.1022)}{25.9284} + \underset{(3.7275)}{0.8082P_{M,t-1}} - \underset{(-0.9482)}{0.1853P_{M,t-2}}$$

$$- \underset{(-0.5406)}{0.0140}(S_{M,t-1} + WQ^{SCRAP}_{M,t-1} - D_{M,t-1})$$

$$+ \underset{(3.1869)}{5.1756r_t} + \underset{(2.1822)}{0.5195P_{R,t}} - \underset{(-3.1030)}{0.7244P_{R,t-1}}$$

$$+ \underset{(0.0429)}{0.8189Y_t}$$

$$R^2 = 0.9638 \qquad SEE = 3.8952$$

We again changed our inventory variable to include scrap. Further, we eliminated some variables reported in Table 8-8 that proved unsatisfactory in the context of the simulation model.

Using this model, we performed an in-sample prediction for the period 1965-1977. This simulation is presented as Simulation #00. These values can be compared with the actual values presented in Tables 8-1 through 8-4.

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[5] The dummy variable is equal to one in 1963-1976 and zero otherwise.

[6] The dummy variable is equal to one in 1972-1976 and zero otherwise.

[7] The dummy variable is equal to one in 1970-1976 and zero otherwise.

[8] The dummy variable is equal to one in 1968-1976 and zero otherwise.

[9] See footnote 5.

[10] The dummy variable is equal to one in 1975-1976 and zero otherwise.

[11] See footnote 5.

TABLE 10-4
SIMULATION EQUATIONS
PRIMARY PRODUCTION
NICKEL
(T-STATISTICS IN PARENTHESES)

		North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
θ_0	Constant	-857.182 (-1.1690)	84.8718 (0.6417)		75.7832 (4.2018)		-32.4291 (-0.1727)	-80.4032 (-1.3042)
θ_1	$P_{M,t}$	1.3088 (0.4487)					1.4465 (0.7011)	1.4350 (3.0462)
θ_2	$P_{M,t-1}$	-0.4006 (-0.2417)	0.2931 (0.7832)		0.3381 (2.8395)		-0.1159 (-0.1389)	-1.0362 (-2.9882)
θ_3	$ROWS_{M,t-1}$	-0.3506 (-0.5593)	-0.0698 (-1.4467)		-0.0049 (-0.3899)		-0.0219 (-0.0800)	0.1397 (2.6337)
θ_4	$D_{M,t}$	0.2007 (0.7790)	0.0757 (1.3019)		0.0328 (2.3078)		0.1373 (0.8640)	0.0555 (0.8576)
θ_5	$D_{M,t-1}$	0.3220 (1.7191)	0.0811 (1.2357)		0.0353 (1.5477)		0.1978 (1.9718)	-0.0205 (-0.4220)
θ_6	t	17.2693 (1.0703)	-2.0665 (-0.6873)		-2.0038 (-4.2930)		-3.4541 (-0.5651)	0.9874 (0.7320)
θ_7	r_t		NA		NA		11.3332 (1.4397)	NA
θ_8	r_{t-1}	10.5975 (0.4740)	NA		NA		9.3204 (0.6719)	NA
θ_9	$\overset{t-2}{\Sigma} Q^S_{M,i}$	-0.0873 (-0.737[illegible]	0.0592 (0.5823)		0.2845 (6.6761)		-0.0364 (-1.2312)	-0.0492 (-2.8258)
θ_{10}	$Q^S_{M,t-1}$	-0.2648 (1.0763)	0.3593 (2.4180)		-0.6650 (-2.3848)		-0.1474 (-0.5944)	0.4289 (3.0496)
	Dummy 1		-21.4206 (-2.7258)		7.3781 (2.5837)		38.7820 (1.0715)	37.0951 (4.8074)
	Dummy 2		34.2518 (4.5041)					
	R^2	0.8703	0.9474		0.9918		0.9886	0.9865
	SEE	25.1435	6.5956		1.5668		9.6666	6.5040

TABLE 10-5
SIMULATION EQUATIONS
DEMAND
NICKEL
(T-STATISTICS IN PARENTHESES)

		North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
Ψ_0	Constant	-90.9912 (-2.1983)			-2.1142 (-0.5566)	-122.367 (-4.5159)	-9.2509 (-6.6680)	4.9018 (0.8933)
Ψ_1	$P_{M,t}-P_{R,t}$							-0.3387 (-1.1584)
Ψ_2	$P_{M,t-1}-P_{R,t-1}$	-0.8062 (-1.2435)				-0.4728 (-1.0191)	-0.0222 (-1.0658)	-0.2333 (-0.8251)
Ψ_3	$S_{M,t-1}-D_{M,t-1}$	-0.1138 (-1.2219)				-0.0792 (-1.1374)	-0.0023 (-0.8358)	0.0116 (0.3796)
Ψ_4	Y_t	294.110 (2.2826)			1.3058 (0.1302)	374.079 (2.1062)	7.1330 (1.3136)	-3.9232 (-0.0423)
Ψ_5	Y_{t-1}	-26.5808 (-0.1654)			2.0994 (0.2195)	-51.5532 (-0.2514)	7.8341 (1.2450)	52.3432 (0.5300)
Ψ_6	$\dot{K}_{t-1}$	59.8842 (0.7781)				9.9632 (0.1380)	1.4668 (0.5303)	NA
Ψ_7	$\dot{M}_{t-1}$	0.5157 (0.2585)			NA	-0.5350 (-0.5299)	0.0159 (0.8428)	NA
Ψ_8	$Q^D_{M,t-1}$	-0.0431 (-0.1854)			0.7314 (3.2301)	-0.2065 (-0.8248)	-0.4507 (-2.6534)	0.5419 (2.8997)
	Dummy	-24.0216 (-1.4907)				-23.8646 (-1.6657)	-2.3437 (-4.2755)	13.3891 (1.8258)
	R^2	0.9134			0.8577	0.9759	0.9400	0.9852
	SEE	12.0044			6.3311	9.6802	0.4344	5.5125

We then proceeded to simulate the future values of production, consumption, and price for nickel. As described in section 10.2, these simulations are presented as Simulation #0 through Simulation #6. The rate of change of the price of steel — the related commodity — over the period 1962-1976 was 0.021.

Even though our in-sample predictions proved to be quite satisfactory, the future simulations proved to be somewhat disquieting — particularly in the case of primary production in North America. As the reader will note, our predictions indicate that North American production will peak in the early 1980's and decline thereafter. Although this situation is plausible, we felt that it deserved some additional consideration.

As we noted in Chapter 8, we were made aware of the assertion that the nickel market is one in which there has existed a dominant firm price leader and that the price leader is located in North America. Indeed, our econometric estimates were consistent with this contention. In such a situation, our future simulations are more understandable. Faced with increased production in the rest of the world, the profit-maximizing dominant firm will reduce output.[12] Thus, in the context of our future simulations, if production in the rest of the country groups increases (as is projected), production in North America would be expected to decline.

However, with this increased production in the rest of the world, we would expect the market power of the dominant firm to decline, thereby reducing its ability to set price. It would then be most useful to have simulations of the future conditions in the nickel market, assuming that the market evolves into a more competitive structure. Obviously, the problem exists that we can not estimate such an alternative structure, since the available historical data incorporates the price leadership behavior. We can, however, use some accepted theoretical principles to modify our estimated equations.

We expect the influences of a price leader to appear in two equations, the supply function for the dominant country group (North America) and the price equation. Let us begin with the primary production equation for North America. As we have argued earlier, the dominant firm will be more responsive to supply from the rest of the world than would be the other suppliers. This is confirmed in our estimates by comparing the estimate of θ_3 in North America with those for the other country groups. Furthermore, as we noted in Chapter 8, it is only the dominant firm that will be concerned with world (market) demand. This is supported by the fact that the estimates of θ_4 and θ_5 are larger for North America than for the other country groups. Therefore, to indicate an elimination of the price leadership behavior, the coefficients θ_3, θ_4, and θ_5 must be altered. In our modified simulation, we have assumed that North America will behave more like Oceania (another large producer of nickel) in the future. Hence, we employed in our modified primary production equation for North America, the Oceania estimates for θ_3, θ_4, and θ_5.[13]

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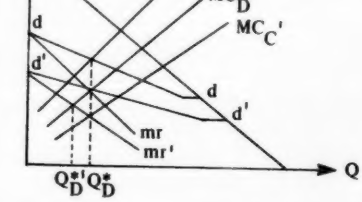
[12] To illustrate this, let us return to the discussion contained in footnote 2 to Chapter 8. Begin with the equilibrium described there, where the dominant firm is producing Q_D^*. Now, if production from the competitive fringe increases, this would entail a shift in the marginal cost curve for the competitive fringe from MC_C to MC_C'. As a result of this shift, the dominant firm's "left over" demand curve shifts from dd to d'd'. It follows then that production from the dominant firm will decline to $Q_D^{*\prime}$.



[13] Note that these values are within the 95% confidence interval of the original estimates for North America.

NICKEL
SIMULATION # 00

Year	North America Supply	North America Demand	South America Supply	South America Demand	Asia Supply	Asia Demand	Africa Supply	Africa Demand	Europe Supply	Europe Demand	Oceania Supply	Oceania Demand	Centrally Planned Supply	Centrally Planned Demand
1965	249.0	148.4	35.4	1.7	1.9	42.3	14.3	1.7	4.3	121.9	103.3	2.8	118.5	100.3
1966	210.4	140.1	40.8	2.2	2.5	49.9	7.9	2.2	4.4	118.8	97.7	2.7	134.4	99.6
1967	222.0	151.5	47.5	2.8	3.2	56.3	15.4	2.7	5.2	131.4	102.3	3.2	133.4	101.7
1968	231.6	159.1	53.5	3.4	4.0	62.3	12.9	3.1	6.1	139.0	150.9	3.3	134.0	105.2
1969	224.3	161.7	56.5	4.1	5.0	68.5	18.8	3.4	6.9	144.2	151.9	3.6	138.8	109.9
1970	235.2	170.4	62.8	4.7	6.2	74.9	26.2	3.7	8.2	154.2	159.8	3.9	138.6	114.7
1971	242.5	177.0	69.6	5.4	7.7	81.5	27.3	4.0	9.7	162.2	168.6	4.1	139.8	119.7
1972	250.4	184.0	111.6	6.1	9.5	88.5	34.7	4.3	11.3	171.0	177.2	4.4	140.7	124.6
1973	241.6	186.5	131.6	6.8	11.6	95.8	37.9	4.5	13.0	176.7	183.8	4.6	146.1	130.3
1974	244.9	193.4	147.6	7.6	14.1	103.4	46.2	4.8	14.9	186.3	191.8	5.0	148.5	136.2
1975	241.7	175.9	160.0	8.4	17.8	111.4	51.2	5.0	16.1	171.2	196.4	2.9	149.4	155.7
1976	241.6	181.3	173.9	9.2	21.7	119.8	61.1	5.3	18.3	183.8	201.2	4.2	150.4	169.6
1977	254.1	190.3	192.1	10.0	26.3	128.5	70.7	5.5	20.9	194.1	215.8	4.1	152.6	179.9

SECONDARY PRODUCTION

Year	North America
1965	18.7
1966	16.2
1967	15.6
1968	15.0
1969	14.8
1970	15.0
1971	15.2
1972	15.7
1973	16.0
1974	16.7
1975	16.6
1976	17.9
1977	18.8

WORLD

Year	Production	Consumption	Price	Scrap
1965	526.8	418.9	85.486	18.7
1966	498.1	415.5	85.624	16.2
1967	528.9	449.5	86.617	15.6
1968	593.0	475.3	88.376	15.0
1969	602.2	495.3	90.070	14.8
1970	637.0	526.5	92.313	15.0
1971	665.0	553.9	94.866	15.2
1972	735.3	583.0	97.676	15.7
1973	765.6	605.3	100.134	16.0
1974	808.1	636.7	102.800	16.7
1975	832.6	630.5	105.729	16.6
1976	868.1	673.1	108.651	17.9
1977	932.6	712.5	112.113	18.8

NICKEL
SIMULATION # 0

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	273.9	199.0	107.0	13.0	67.9	159.6	88.8	6.1	35.4	218.8	260.5	4.5	151.2	192.2
1981	272.8	204.3	117.2	14.0	82.3	170.0	99.9	6.4	39.2	227.9	274.3	4.7	159.4	200.7
1982	272.8	209.8	128.4	14.9	99.9	181.1	119.4	6.6	43.4	237.9	288.3	5.1	167.2	209.3
1983	268.7	214.0	139.9	15.9	121.3	192.7	136.5	6.9	47.9	247.2	302.5	5.3	176.2	218.1
1984	264.9	218.4	152.2	16.9	147.2	204.9	160.8	7.2	52.9	256.9	317.0	5.7	185.3	227.3
1985	257.5	221.6	164.8	18.0	178.6	217.6	185.3	7.4	58.1	266.1	331.7	5.9	195.7	237.1
1986	249.1	224.6	178.0	19.1	216.7	231.0	216.6	7.7	63.8	275.5	346.6	6.2	206.8	247.5
1987	236.8	226.3	191.3	20.2	263.0	245.0	250.4	8.0	69.8	284.4	361.6	6.5	219.4	258.7
1988	222.3	227.4	204.9	21.3	319.0	259.6	291.4	8.3	76.2	293.2	376.6	6.8	233.4	270.8
1989	203.3	227.1	218.2	22.5	387.1	275.0	337.3	8.6	83.0	301.4	391.7	7.0	249.3	283.9
1990	180.4	225.5	231.3	23.7	469.5	291.0	391.5	8.9	90.2	309.2	406.5	7.3	267.0	298.3
1991	151.8	222.2	243.6	25.0	569.6	307.8	453.1	9.2	97.8	316.1	421.1	7.5	287.2	314.0

SECONDARY PRODUCTION

Year	North America
1980	24.3
1981	25.2
1982	26.2
1983	27.2
1984	28.3
1985	29.4
1986	30.6
1987	31.9
1988	33.1
1989	34.4
1990	35.7
1991	36.9

WORLD

Year	Production	Consumption	Price	Scrap
1980	984.7	793.1	131.996	24.3
1981	1045.0	828.0	136.012	25.2
1982	1119.4	864.7	140.412	26.2
1983	1193.0	900.1	144.979	27.2
1984	1280.3	937.2	149.731	28.3
1985	1371.6	973.7	154.568	29.4
1986	1477.5	1011.5	159.515	30.6
1987	1592.3	1049.0	164.500	31.9
1988	1723.9	1087.4	169.524	33.1
1989	1869.7	1125.5	174.515	34.4
1990	2036.5	1163.9	179.442	35.7
1991	2224.1	1201.8	184.221	36.9

NICKEL
SIMULATION # 1

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	301.3	240.7	124.3	15.6	64.2	199.8	94.4	6.7	40.6	249.0	291.1	5.6	170.8	196.5
1981	305.2	257.2	140.3	17.2	77.0	216.9	107.3	7.2	46.3	266.8	312.5	6.2	184.7	207.6
1982	310.6	274.8	158.1	18.9	92.6	235.2	129.2	7.8	52.5	285.9	334.7	6.9	198.2	219.2
1983	312.2	291.9	177.2	20.7	111.4	254.5	149.2	8.4	59.3	305.0	357.5	7.6	212.9	231.4
1984	314.4	310.3	198.4	22.5	134.1	275.2	176.9	9.1	66.8	325.4	381.3	8.3	227.9	244.4
1985	313.2	328.5	221.0	24.5	161.6	297.2	205.4	9.8	74.9	346.1	405.8	9.0	244.5	258.4
1986	311.5	347.6	245.7	26.5	194.9	320.6	241.6	10.5	83.7	367.8	431.3	9.8	262.1	273.6
1987	306.3	366.7	272.0	28.7	235.2	345.7	281.0	11.2	93.3	389.9	457.5	10.5	281.5	290.1
1988	299.1	386.4	300.4	30.9	283.9	372.3	328.8	12.0	103.6	412.9	484.7	11.4	302.6	308.1
1989	287.8	406.0	330.5	33.3	342.8	400.7	382.6	12.9	114.8	436.3	512.5	12.2	326.0	327.7
1990	273.2	425.9	362.5	35.7	414.2	431.0	446.2	13.7	126.8	460.5	541.2	13.1	351.7	349.3
1991	253.3	445.4	396.0	38.3	500.7	463.3	518.8	14.6	139.7	484.9	470.4	14.0	380.4	373.0

SECONDARY PRODUCTION

Year	North America
1980	25.0
1981	25.8
1982	26.6
1983	27.5
1984	28.5
1985	29.5
1986	30.6
1987	31.7
1988	32.8
1989	33.9
1990	35.0
1991	36.1

WORLD

Year	Production	Consumption	Price	Scrap
1980	1086.7	913.9	132.676	25.0
1981	1173.2	979.1	136.868	25.8
1982	1275.8	1048.6	141.477	26.6
1983	1379.7	1119.5	146.286	27.5
1984	1499.7	1195.2	151.311	28.5
1985	1626.5	1273.4	156.454	29.5
1986	1770.7	1356.5	161.737	30.6
1987	1926.8	1442.9	167.089	31.7
1988	2103.1	1534.1	172.512	32.8
1989	2297.1	1629.2	177.934	33.9
1990	2515.8	1729.2	183.328	35.0
1991	2759.3	1833.5	188.611	36.1

NICKEL
SIMULATION #2

Year	North America Supply	North America Demand	South America Supply	South America Demand	Asia Supply	Asia Demand	Africa Supply	Africa Demand	Europe Supply	Europe Demand	Oceania Supply	Oceania Demand	Centrally Planned Supply	Centrally Planned Demand
1980	248.3	160.4	90.6	10.6	71.4	121.6	83.6	5.5	30.3	190.2	231.5	3.5	132.5	187.9
1981	243.4	156.4	95.7	10.9	87.4	126.6	92.9	5.6	32.6	192.0	239.0	3.4	135.6	194.1
1982	239.3	152.6	101.2	11.3	106.8	132.1	110.3	5.5	35.0	194.4	246.4	3.4	138.8	199.9
1983	231.2	147.1	106.2	11.6	130.4	137.8	125.0	5.5	37.7	195.6	253.9	3.4	143.3	205.7
1984	223.3	141.5	111.3	12.0	159.0	143.8	146.5	5.5	40.6	197.0	261.5	3.4	148.0	211.5
1985	211.7	134.4	115.7	12.4	193.8	149.9	167.6	5.4	43.6	197.4	269.0	3.3	154.1	217.6
1986	199.1	126.9	119.8	12.7	236.1	156.2	194.9	5.4	46.8	197.7	276.6	3.3	160.8	224.0
1987	182.7	117.7	123.0	13.1	287.4	162.6	224.1	5.3	50.1	197.1	284.0	3.2	169.2	230.9
1988	164.0	107.6	125.3	13.5	349.6	169.3	259.7	5.2	53.6	196.0	291.4	3.1	178.8	238.4
1989	140.7	95.7	126.1	13.9	425.2	176.1	299.2	5.1	57.3	193.8	298.4	3.0	190.2	246.5
1990	113.5	82.3	125.3	14.2	516.9	183.2	346.1	5.1	61.0	190.7	305.0	2.9	203.5	255.3
1991	80.4	66.7	122.0	14.6	628.2	190.4	399.0	5.0	64.9	186.4	311.0	2.7	219.2	265.1

SECONDARY PRODUCTION

Year	North America
1980	23.7
1981	24.8
1982	25.9
1983	27.0
1984	28.3
1985	29.5
1986	30.9
1987	32.2
1988	33.7
1989	35.1
1990	36.5
1991	38.0

WORLD

Year	Production	Consumption	Price	Scrap
1980	888.3	679.7	131.352	23.7
1981	926.5	689.0	135.220	24.8
1982	977.9	699.2	139.449	25.9
1983	1027.7	706.7	143.824	27.0
1984	1090.1	714.6	148.362	28.3
1985	1155.5	720.5	152.967	29.5
1986	1234.2	726.2	157.667	30.9
1987	1320.6	730.0	162.390	32.2
1988	1422.4	733.0	167.138	33.7
1989	1537.1	734.1	171.839	35.1
1990	1671.2	733.7	176.463	36.5
1991	1824.7	730.9	180.923	38.0

NICKEL
SIMULATION # 3

	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
Year	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	266.6	198.7	102.1	13.0	69.0	127.0	87.3	6.1	33.9	218.6	251.8	4.5	145.4	192.4
1981	265.4	203.9	111.1	14.0	83.8	133.2	97.9	6.4	37.3	227.7	264.7	4.7	152.5	201.0
1982	265.0	209.4	121.2	14.9	101.9	139.9	117.0	6.6	41.2	237.7	277.8	5.1	159.7	209.6
1983	260.4	213.5	131.3	15.9	123.8	147.1	133.5	6.9	45.3	246.8	291.0	5.3	168.2	218.4
1984	256.1	217.8	142.1	16.9	150.3	154.4	157.2	7.2	49.9	256.6	304.5	5.6	176.7	227.7
1985	248.1	220.9	153.0	18.0	182.6	162.1	181.0	7.4	54.7	265.7	318.0	5.9	186.6	237.5
1986	239.0	223.8	164.3	19.1	221.7	170.0	211.4	7.7	59.9	275.1	331.8	6.2	197.2	247.9
1987	226.2	225.5	175.6	20.2	269.1	178.2	244.1	8.0	65.4	283.9	345.5	6.5	209.3	259.2
1988	211.0	226.5	186.8	21.3	326.7	186.6	284.0	8.3	71.2	292.6	359.3	6.8	222.6	271.4
1989	191.2	226.1	197.6	22.5	396.5	195.3	328.4	8.6	77.4	300.7	373.0	7.0	237.8	284.6
1990	167.6	224.4	207.8	23.7	481.3	204.3	381.0	8.9	84.0	308.5	386.4	7.3	254.8	299.0
1991	138.1	221.0	216.8	25.0	584.0	213.6	440.7	9.2	90.8	315.4	399.4	7.5	274.3	314.8

SECONDARY PRODUCTION

Year	North America
1980	24.2
1981	25.2
1982	26.2
1983	27.3
1984	28.4
1985	29.6
1986	30.8
1987	32.1
1988	33.3
1989	34.6
1990	36.0
1991	37.2

WORLD

Year	Production	Consumption	Price	Scrap
1980	956.0	760.3	131.852	24.2
1981	1012.9	790.8	135.863	25.2
1982	1083.6	823.2	140.249	26.2
1983	1153.5	854.0	144.794	27.3
1984	1236.8	886.2	149.518	28.4
1985	1323.9	917.5	154.323	29.6
1986	1425.3	949.8	159.235	30.8
1987	1535.3	981.4	164.184	32.1
1988	1661.7	1013.5	169.170	33.3
1989	1802.0	1044.8	174.120	34.6
1990	1962.8	1076.1	179.006	36.0
1991	2144.1	1106.4	183.740	37.2

NICKEL
SIMULATION # 4

	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
Year	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	263.1	198.6	99.7	13.1	69.5	118.6	86.5	6.1	33.1	211.0	247.7	4.5	142.7	192.5
1981	261.4	203.7	108.1	14.0	84.6	123.5	96.9	6.4	36.4	218.1	259.8	4.7	149.2	201.1
1982	260.4	209.1	117.3	14.9	102.9	128.9	115.7	6.6	40.0	225.9	272.0	5.1	155.8	209.7
1983	255.4	213.2	126.6	15.9	125.1	134.6	131.9	6.9	43.9	232.9	284.4	5.3	163.7	218.6
1984	250.5	217.4	136.4	16.9	152.0	140.6	155.2	7.2	48.2	240.2	297.0	5.6	171.7	227.9
1985	241.9	220.5	146.2	18.0	184.7	146.7	178.5	7.4	52.7	246.8	309.6	5.9	181.1	237.7
1986	232.3	223.4	156.3	19.1	224.4	153.0	208.4	7.7	57.6	253.5	322.4	6.2	191.1	248.2
1987	218.9	225.0	166.2	20.2	272.6	159.5	240.5	8.0	62.7	259.5	335.2	6.5	202.6	259.6
1988	203.1	225.9	175.9	21.3	331.0	166.1	279.6	8.3	68.2	265.4	347.9	6.7	215.4	271.8
1989	182.8	225.4	185.0	22.5	401.9	173.0	323.2	8.6	74.0	270.4	360.5	7.0	230.0	285.1
1990	158.6	223.7	193.3	23.7	487.9	180.0	374.8	8.9	80.0	274.8	372.9	7.3	246.4	299.5
1991	128.5	220.2	200.2	25.0	592.3	187.2	433.2	9.2	86.4	278.3	384.7	7.5	265.3	315.4

SECONDARY PRODUCTION

Year	North America
1980	24.1
1981	25.1
1982	26.2
1983	27.3
1984	28.4
1985	29.6
1986	30.9
1987	32.1
1988	33.4
1989	34.8
1990	36.1
1991	37.4

WORLD

Year	Production	Consumption	Price	Scrap
1980	942.3	744.2	131.787	24.1
1981	996.4	771.4	135.788	25.1
1982	1064.2	800.3	140.160	26.2
1983	1131.0	827.5	144.689	27.3
1984	1211.1	855.8	149.394	28.4
1985	1294.8	883.1	154.177	29.6
1986	1392.6	911.1	159.066	30.9
1987	1498.8	938.2	163.990	32.1
1988	1621.2	965.5	168.949	33.4
1989	1757.4	991.9	173.871	34.8
1990	1913.8	1017.9	178.727	36.1
1991	2090.6	1042.7	183.430	37.4

NICKEL
SIMULATION # 5

Year	North America Supply	North America Demand	South America Supply	South America Demand	Asia Supply	Asia Demand	Africa Supply	Africa Demand	Europe Supply	Europe Demand	Oceania Supply	Oceania Demand	Centrally Planned Supply	Centrally Planned Demand
1980	274.9	199.3	107.3	13.0	67.9	159.6	85.3	6.1	35.4	218.9	260.6	4.5	150.9	192.1
1981	273.8	204.5	117.6	14.0	82.4	170.0	95.1	6.4	39.2	228.1	274.5	4.8	159.0	200.6
1982	274.1	210.2	128.9	14.9	100.0	181.1	113.5	6.5	43.4	238.2	288.6	5.1	166.7	209.1
1983	270.3	214.4	140.5	15.9	121.3	192.7	129.2	6.9	48.0	247.4	302.9	5.4	175.6	217.9
1984	266.9	218.9	153.1	16.9	147.2	204.9	151.6	7.2	52.9	257.3	317.5	5.7	184.6	227.0
1985	259.9	222.2	165.9	18.0	178.7	217.6	174.1	7.4	58.2	266.5	332.2	5.9	194.8	236.7
1986	252.0	225.3	179.4	19.1	216.8	231.0	202.6	7.7	63.9	276.0	347.2	6.2	205.7	247.1
1987	240.4	227.3	193.1	20.2	263.1	245.0	233.5	8.0	69.9	285.0	362.4	6.5	218.2	258.2
1988	226.6	228.6	207.1	21.3	319.2	259.6	270.6	8.3	76.4	294.0	377.6	6.8	231.9	270.1
1989	208.5	228.5	221.0	22.5	387.2	275.0	312.1	8.6	83.2	302.3	392.8	7.0	247.4	283.2
1990	186.6	227.3	234.7	23.7	469.8	291.0	360.7	8.9	90.5	310.3	408.0	7.3	264.8	297.3
1991	159.3	224.3	247.7	25.0	569.9	307.8	416.0	9.2	98.1	317.5	422.8	7.5	284.5	312.8

SECONDARY PRODUCTION

Year	North America
1980	24.3
1981	25.2
1982	26.2
1983	27.2
1984	28.3
1985	29.5
1986	30.7
1987	31.9
1988	33.2
1989	34.5
1990	35.8
1991	37.1

WORLD

Year	Production	Consumption	Price	Scrap
1980	982.3	793.5	132.060	24.3
1981	1041.6	828.2	136.093	25.2
1982	1115.3	865.1	140.517	26.2
1983	1187.9	900.6	145.113	27.2
1984	1273.7	937.8	149.897	28.3
1985	1363.8	974.4	154.777	29.5
1986	1467.6	1012.4	159.772	30.7
1987	1580.5	1050.1	164.819	31.9
1988	1709.2	1088.7	169.914	33.2
1989	1852.2	1127.1	174.993	34.5
1990	2015.0	1165.8	180.023	35.8
1991	2198.3	1204.1	184.927	37.1

NICKEL
SIMULATION # 6

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	271.2	198.5	106.1	13.0	67.8	159.6	88.7	6.1	35.3	218.4	259.9	4.5	161.8	192.5
1981	270.4	203.7	116.1	14.0	82.3	170.0	99.7	6.4	39.1	227.6	273.7	4.7	169.9	201.1
1982	270.6	209.3	127.2	14.9	99.9	181.1	119.2	6.6	43.3	237.6	287.8	5.1	177.6	209.7
1983	266.3	213.5	138.6	15.9	121.2	192.7	136.3	6.9	47.8	246.8	302.0	5.3	186.4	218.6
1984	263.3	217.8	150.8	16.9	147.1	204.9	160.6	7.2	52.7	256.6	316.6	5.6	195.4	227.8
1985	256.0	221.0	163.2	18.0	178.5	217.6	185.1	7.4	58.0	265.8	331.2	5.9	205.7	237.5
1986	247.8	224.0	176.3	19.1	216.6	231.0	216.4	7.7	63.6	275.2	346.2	6.2	216.8	247.9
1987	235.7	225.8	189.5	20.2	262.8	245.0	250.1	8.0	69.6	284.1	361.2	6.5	229.7	259.2
1988	221.3	226.9	202.9	21.3	318.9	259.6	291.1	8.3	76.1	292.9	376.3	6.8	243.8	271.3
1989	202.3	226.5	216.1	22.5	386.8	275.0	336.9	8.6	82.8	301.0	391.3	7.0	260.0	284.4
1990	179.5	225.0	228.9	23.7	469.3	291.0	391.1	8.9	90.0	308.8	406.2	7.3	278.2	298.8
1991	150.8	221.6	240.9	25.0	569.3	307.8	452.6	9.2	97.6	315.8	420.7	7.5	299.0	314.5

SECONDARY PRODUCTION

Year	North America
1980	24.3
1981	25.2
1982	26.1
1983	27.1
1984	28.2
1985	29.4
1986	30.6
1987	31.8
1988	33.1
1989	34.3
1990	35.6
1991	36.9

WORLD

Year	Production	Consumption	Price	Scrap
1980	990.8	792.5	131.776	24.3
1981	1051.3	827.4	135.773	25.2
1982	1125.6	864.3	140.165	26.1
1983	1199.3	899.7	144.731	27.1
1984	1286.5	936.8	149.483	28.2
1985	1377.8	973.3	154.322	29.4
1986	1483.7	1011.1	159.270	30.6
1987	1598.6	1048.6	164.257	31.8
1988	1730.3	1087.0	169.280	33.1
1989	1876.2	1125.0	174.269	34.3
1990	2043.2	1163.4	179.193	35.6
1991	2231.0	1201.3	183.967	36.9

NICKEL
MODIFIED BASE SIMULATION

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	359.4	221.7	107.7	13.0	64.8	159.6	89.2	6.1	35.6	232.1	261.2	4.9	158.6	199.1
1981	389.5	231.5	116.7	14.0	78.2	170.0	100.3	6.4	39.3	244.1	274.5	5.2	170.2	210.3
1982	423.1	241.4	126.2	14.9	94.3	181.1	119.7	6.6	43.4	256.5	288.1	5.6	182.2	221.8
1983	460.2	251.5	135.6	15.9	113.7	192.7	136.9	6.9	47.8	269.3	301.6	6.0	195.6	233.9
1984	502.0	262.2	145.3	16.9	137.2	204.9	161.0	7.2	52.6	282.7	315.4	6.4	209.6	247.0
1985	549.0	273.4	154.9	18.0	165.5	217.6	185.5	7.4	57.6	296.6	329.3	6.8	225.4	261.2
1986	602.5	285.3	164.5	19.1	199.7	231.0	216.6	7.7	63.1	311.1	343.5	7.2	242.7	276.7
1987	663.4	297.9	173.7	20.2	241.0	245.0	250.3	8.0	68.8	326.3	357.7	7.7	262.2	293.8
1988	733.2	311.3	182.2	21.3	290.8	259.6	291.1	8.3	74.9	342.3	372.0	8.2	284.0	312.8
1989	813.5	325.6	189.7	22.5	351.1	275.0	336.7	8.6	81.4	359.1	386.2	8.7	308.7	333.9
1990	906.5	341.1	195.7	23.7	423.9	291.0	390.6	8.9	88.2	376.8	400.1	9.2	336.7	357.4
1991	1014.3	357.8	199.7	25.0	511.8	307.8	451.9	9.2	95.3	395.5	413.5	9.8	368.7	383.8

SECONDARY PRODUCTION

Year	North America
1980	23.1
1981	23.4
1982	23.7
1983	24.0
1984	24.3
1985	24.6
1986	24.7
1987	24.7
1988	24.6
1989	24.2
1990	23.6
1991	22.7

WORLD

Year	Production	Consumption	Price	Scrap
1980	1076.5	836.5	124.022	23.1
1981	1168.7	881.4	126.057	23.4
1982	1277.0	927.8	128.163	23.7
1983	1391.3	976.2	129.969	24.0
1984	1523.2	1027.1	131.527	24.3
1985	1667.3	1081.0	132.641	24.6
1986	1832.6	1138.1	133.311	24.7
1987	2017.1	1198.9	133.336	24.7
1988	2228.4	1263.8	132.631	24.6
1989	2467.3	1333.3	130.964	24.2
1990	2741.7	1408.1	128.163	23.6
1991	3055.3	1488.9	123.933	22.7

The price equation as estimated is generally satisfactory. However, we would normally expect price to be more sensitive to changes in inventories in a competitive market than in one characterized by price leadership. Therefore, in our modified simulation, we changed this coefficient(η_3) to the one estimated for the copper market. That is, we changed η_3 from -0.0140 to -0.0292.[14]

Before simulating, we made one additional change. As the reader will note from Table 10-5, the sign of the coefficient for the lagged difference between primary production and consumption(Ψ_3)was the opposite of that predicted theoretically for North America, Europe, and Oceania. Since all of the estimated coefficients were statistically insignificant, we set them equal to zero.

We incorporated these modifications into our simulation model and reran the base simulation (Simulation #0). The resulting simulation is presented as the Modified Base Simulation. The differences between this modified simulation and Simulation #0 are apparent.

We must stress the fact that we have presented this modified simulation only as an illustration. While we are relatively sure that the changes made are in the correct direction, there is no way of determining the correct magnitudes of the modified coefficients. However, this procedure can give the reader some idea of the effect on the market if indeed the hypothesized change in structure occurs. Furthermore, this procedure provides another example of the flexibility of our approach, in that we can simulate changes in the underlying structure of the market.

10.6 ZINC

Again, we performed several in-sample predictions. The results of these simulations indicated that modifications were required in primary production, demand, and the price equation.

For primary production, only the equation for Africa was reestimated. The results of this estimation are as follows:

$$Q^S_{M,t} = \underset{(-3.4464)}{-4582.93} + \underset{(1.8274)}{2.7020}P_{M,t-1} - \underset{(-1.6103)}{0.0288}ROWS_{M,t-1} + \underset{(1.4854)}{0.0222}D_{M,t-1}$$

$$+ \underset{(3.5444)}{94.5641}t - \underset{(-3.5181)}{0.3663}\sum^{t-2} Q^S_{M,i} - \underset{(-0.2788)}{0.0614}Q^S_{M,t-1}$$

$$R^2 = 0.6354 \qquad SEE = 16.3408$$

As should be noted, the primary change in this formulation is the elimination of the twice lagged terms for price and world consumption.

In the case of demand, alternative equations were estimated for North America, South America, Asia, Europe, and Oceania. These equations are presented in Table 10-6. In them, the only change made was the inclusion of a dummy variable to reflect the recessionary effects of 1975-1976.[15]

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[14] Note from our estimates that -0.0292 is within the 95% confidence interval of the estimated value of η_3 for nickel.

[15] The dummy variable is equal to one in 1975-1976 and zero otherwise.

The alternative price equation was estimated as follows:

$$
\begin{aligned}
P_{M,t} = {} & \underset{(2.3921)}{84.9847} + \underset{(3.8991)}{0.5423}P_{M,t-1} \underset{(-1.0041)}{-0.2490}P_{M,t-2} \\
& \underset{(-1.3690)}{-\,0.0026}(S_{M,t-1} + WQ^{SCRAP}_{M,t-1} - D_{M,t-1}) \\
& \underset{(-3.6834)}{-\,1.4996}t \;\underset{(-0.8571)}{-\,0.7031}r_t + \underset{(1.9877)}{0.0005}\sum^{t-2} S_{M,i} \\
& + \underset{(0.9366)}{0.1216}P_{R,t} \underset{(-2.8532)}{-\,0.2720}P_{R,t-1} + \underset{(2.5436)}{0.1989}P_{R,t-2} \\
& \underset{(-0.0381)}{-\,0.5750}Y_t \\
& R^2 = 0.9208 \qquad SEE = 1.7332
\end{aligned}
$$

Comparing these estimates with those presented in Table 9-8, the only differences are the reformulation of the inventory variable to include scrape and the inclusion of the current price of the related commodity (steel).

Using this modified simulation model, we simulated production, consumption, and price for the period 1965-1977. These are presented as Simulation #00. The reader should compare these values with those displayed in Tables 9-1 through 9-4.

Finally, the simulations of the future values of production, consumption, and price are presented in Simulation #0 through Simulation #6. We used the growth rates for the country specific variables as presented in section 10.2. The mean rate of change in the price of steel was calculated as +0.021.

ZINC
SIMULATION # 00

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1965	1311.0	1190.2	543.1	117.0	272.1	524.0	272.0	47.0	493.9	1282.7	380.3	101.8	840.4	739.3
1966	1404.0	1216.6	567.3	127.7	294.3	610.2	259.2	55.5	506.0	1285.1	403.2	105.5	910.5	794.4
1967	1460.0	1241.7	594.8	139.6	315.8	691.2	250.5	62.0	529.7	1336.6	421.3	110.0	973.4	846.9
1968	1516.5	1271.1	623.0	151.9	337.1	752.3	250.1	67.3	556.6	1384.1	439.4	114.5	1036.2	899.3
1969	1591.3	1303.1	654.0	163.8	360.0	806.7	253.7	72.5	586.2	1427.3	458.7	118.3	1102.7	953.2
1970	1680.6	1335.3	689.3	175.8	384.9	865.6	256.7	77.9	617.8	1468.6	479.2	121.8	1173.8	1009.2
1971	1773.9	1366.6	729.6	188.2	411.3	932.8	257.6	83.6	650.2	1510.8	499.9	125.4	1248.6	1067.9
1972	1866.3	1397.2	775.4	200.9	438.4	1005.9	257.1	89.5	682.1	1554.9	520.2	129.2	1326.6	1129.7
1973	1959.8	1427.9	826.9	214.1	466.0	1081.4	256.8	95.5	712.5	1600.1	540.3	133.0	1408.0	1194.8
1974	2058.2	1459.1	884.7	227.5	494.4	1158.4	257.1	101.6	740.7	1645.6	560.8	136.7	1493.4	1263.5
1975	1888.6	1243.2	874.5	201.3	520.5	933.4	257.8	107.6	715.3	1453.4	528.2	104.1	1543.5	1336.1
1976	1703.5	1108.5	920.0	229.8	536.9	935.0	253.6	121.9	746.4	1503.5	471.0	115.3	1554.4	1412.6
1977	1591.1	1057.5	973.7	240.8	535.4	929.1	262.3	121.7	760.0	1571.0	443.8	120.2	1567.1	1493.5

WORLD

Year	Production	Consumption	Price	Scrap
1965	4112.9	4001.9	13.825	786.2
1966	4344.6	4195.0	13.201	800.5
1967	4545.6	4428.0	13.316	821.3
1968	4759.0	4640.5	14.139	840.4
1969	5006.8	4844.7	15.173	860.8
1970	5282.4	5054.2	16.131	883.5
1971	5571.1	5275.3	17.046	908.4
1972	5866.1	5507.2	18.077	935.1
1973	6170.3	5746.8	19.323	963.4
1974	6489.4	5992.5	20.788	993.4
1975	6328.5	5379.0	22.432	890.4
1976	6185.7	5426.6	23.628	922.7
1977	6133.4	5533.7	25.764	957.8

ZINC
SIMULATION # 0

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	1855.6	1072.3	1173.5	299.5	521.8	981.9	275.8	129.9	769.8	1717.5	542.6	127.3	1669.5	1768.9
1981	2018.9	1097.7	1274.8	316.3	543.1	1077.4	264.9	136.9	784.4	1772.3	576.9	131.6	1757.2	1870.0
1982	2181.5	1122.4	1389.5	333.5	571.8	1186.5	256.4	144.8	804.9	1830.8	613.7	136.1	1854.3	1976.6
1983	2359.8	1151.3	1518.4	351.0	603.8	1303.0	252.8	153.0	831.6	1887.9	653.9	140.2	1962.9	2089.1
1984	2560.6	1180.2	1663.3	368.5	642.6	1431.6	251.8	161.9	865.2	1942.0	698.0	143.8	2084.5	2207.9
1985	2779.2	1206.6	1826.5	386.1	688.8	1577.8	250.9	171.5	906.4	1994.1	745.2	147.1	2219.1	2333.3
1986	3008.6	1229.1	2010.0	404.0	742.2	1743.6	249.2	182.1	955.1	2045.5	794.6	150.0	2365.7	2465.7
1987	3245.9	1247.5	2215.9	422.3	802.3	1927.4	247.0	193.5	1010.5	2096.6	845.6	152.8	2524.2	2605.5
1988	3492.6	1261.9	2446.7	440.8	869.0	2127.8	244.9	205.6	1071.8	2146.8	898.4	155.3	2694.6	2753.2
1989	3751.7	1272.0	2704.9	459.5	942.3	2344.7	243.3	218.4	1138.2	2195.4	953.2	157.4	2877.8	2909.1
1990	4025.4	1277.7	2993.8	478.4	1022.3	2579.2	242.0	232.0	1209.3	2241.9	1010.5	159.1	3074.6	3073.7
1991	4314.8	1278.2	3316.9	497.6	1109.0	2833.1	240.5	246.4	1284.8	2286.1	1070.6	160.2	3285.8	3247.6

WORLD

Year	Production	Consumption	Price	Scrap
1980	6808.6	6097.5	30.489	1064.5
1981	7222.1	6400.0	31.540	1108.6
1982	7672.0	6730.6	33.068	1152.5
1983	8183.1	7075.7	35.049	1197.0
1984	8766.1	7435.9	37.242	1243.5
1985	9416.1	7816.5	39.523	1292.7
1986	10125.4	8220.1	41.942	1345.1
1987	10891.4	8645.6	44.618	1400.7
1988	11718.0	9091.3	47.633	1459.7
1989	12611.5	9556.5	51.016	1522.5
1990	13577.9	10041.9	54.771	1589.6
1991	14622.5	10549.2	58.911	1661.4

ZINC
SIMULATION # 1

Year	North America Supply	North America Demand	South America Supply	South America Demand	Asia Supply	Asia Demand	Africa Supply	Africa Demand	Europe Supply	Europe Demand	Oceania Supply	Oceania Demand	Centrally Planned Supply	Centrally Planned Demand
1980	2319.9	1417.9	1256.3	338.9	554.0	1071.1	272.2	146.1	807.9	1893.2	646.8	140.4	1796.0	1871.6
1981	2617.1	1508.4	1386.2	367.4	598.9	1214.7	257.8	160.3	838.8	1983.8	710.0	148.1	1938.2	2010.2
1982	2912.4	1604.2	1534.2	397.2	651.0	1376.8	247.3	175.8	879.0	2081.2	775.1	156.4	2095.8	2159.1
1983	3228.4	1705.9	1701.5	428.1	711.8	1549.3	243.2	192.0	928.0	2179.2	843.9	164.5	2271.5	2319.0
1984	3575.8	1811.8	1890.7	459.9	782.6	1737.5	242.8	209.1	986.0	2275.8	917.5	172.2	2467.7	2490.5
1985	3951.3	1919.8	2104.9	492.7	863.8	1948.7	242.6	227.3	1053.5	2372.4	995.4	179.6	2684.8	2674.5
1986	4347.9	2029.0	2347.3	526.7	955.0	2185.3	241.3	246.9	1129.9	2470.8	1076.9	187.0	2922.6	2872.0
1987	4763.4	2139.4	2620.7	562.1	1055.7	2446.2	239.3	267.8	1214.5	2571.4	1161.7	194.3	3181.1	3083.9
1988	5200.4	2251.6	2928.8	598.9	1165.9	2730.1	237.4	289.9	1306.4	2673.8	1250.0	201.6	3461.3	3311.3
1989	5663.6	2365.7	3275.4	637.0	1285.6	3037.4	236.0	313.1	1404.7	2777.3	1342.7	208.7	3764.7	3555.3
1990	6156.7	2481.7	3665.1	676.6	1415.4	3370.1	234.8	337.7	1509.5	2881.4	1440.4	215.5	4093.1	3817.1
1991	6682.6	2599.2	4103.1	717.5	1555.6	3731.1	233.4	363.7	1621.0	2986.1	1543.7	222.0	4448.1	4098.0

WORLD

Year	Production	Consumption	Price	Scrap
1980	7653.0	6879.1	30.525	1153.7
1981	8347.0	7393.0	31.546	1216.7
1982	9094.7	7950.8	33.250	1281.2
1983	9928.2	8538.0	35.631	1348.6
1984	10863.0	9156.7	38.410	1420.7
1985	11896.3	9815.0	41.440	1498.6
1986	13020.9	10517.7	44.784	1582.9
1987	14236.4	11265.3	48.592	1674.0
1988	15550.1	12057.2	52.976	1772.2
1989	16972.7	12894.5	57.980	1878.4
1990	18515.0	13780.0	63.623	1993.1
1991	20187.5	14717.6	69.933	2117.2

ZINC
SIMULATION # 2

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	1418.2	765.2	1096.1	262.3	490.6	891.6	279.5	115.0	734.3	1552.5	444.5	115.0	1548.8	1670.7
1981	1468.9	743.8	1172.5	268.7	493.6	939.6	271.9	115.7	734.4	1577.3	454.6	116.3	1587.8	1737.9
1982	1526.2	728.0	1259.0	275.4	497.4	998.4	265.1	117.2	737.8	1604.5	469.2	117.7	1632.8	1807.3
1983	1600.5	714.8	1356.2	281.8	504.3	1063.8	261.4	119.2	745.5	1629.9	488.3	118.8	1685.4	1879.2
1984	1694.6	701.5	1465.2	287.9	516.1	1139.8	259.5	121.8	759.1	1652.1	511.6	119.4	1746.6	1953.8
1985	1802.8	685.4	1587.8	293.9	533.8	1230.9	257.6	125.2	779.6	1672.1	538.2	119.6	1816.1	2031.3
1986	1918.8	665.1	1725.3	299.7	557.3	1338.4	255.0	129.5	807.2	1690.8	566.9	119.6	1893.1	2111.8
1987	2039.6	640.1	1879.3	305.3	586.5	1460.9	252.2	134.6	841.5	1708.5	597.1	119.4	1977.0	2195.4
1988	2165.9	610.5	2051.3	310.9	621.4	1597.0	249.7	140.4	881.6	1724.7	628.8	119.0	2067.9	2282.2
1989	2299.6	576.1	2243.5	316.2	661.8	1746.4	247.6	146.8	926.8	1738.7	662.1	118.2	2166.2	2372.4
1990	2442.2	536.6	2457.9	321.3	707.7	1909.6	245.7	154.0	976.3	1750.2	697.1	116.9	2272.3	2466.1
1991	2594.1	491.5	2697.1	326.2	759.1	2087.8	243.8	162.0	1029.7	1759.1	733.9	115.3	2386.6	2563.5

WORLD

Year	Production	Consumption	Price	Scrap
1980	6012.0	5372.3	30.462	982.7
1981	6183.8	5499.4	31.543	1012.1
1982	6387.5	5648.4	32.896	1040.3
1983	6641.6	5807.5	34.484	1067.8
1984	6952.7	5976.3	36.117	1095.6
1985	7315.8	6158.4	37.700	1124.4
1986	7723.5	6354.9	39.280	1154.3
1987	8173.1	6564.3	40.952	1185.3
1988	8666.5	6784.6	42.780	1217.8
1989	9207.5	7014.8	44.784	1251.8
1990	9799.1	7254.9	46.967	1287.7
1991	10444.3	7505.5	49.340	1325.8

ZINC
SIMULATION # 3

Year	North America Supply	North America Demand	South America Supply	South America Demand	Asia Supply	Asia Demand	Africa Supply	Africa Demand	Europe Supply	Europe Demand	Oceania Supply	Oceania Demand	Centrally Planned Supply	Centrally Planned Demand
1980	1791.4	1070.7	1162.5	299.5	518.5	874.9	275.9	130.2	764.9	1717.0	528.0	127.3	1653.6	1768.9
1981	1934.6	1094.9	1260.3	316.3	538.4	944.6	265.8	136.9	777.6	1772.9	558.0	131.7	1733.2	1870.0
1982	2081.6	1123.2	1371.1	333.7	561.2	1027.4	258.0	144.4	795.5	1832.2	591.6	136.3	1822.1	1976.6
1983	2247.3	1154.1	1495.7	351.3	589.2	1117.5	254.5	152.3	819.2	1890.0	629.3	140.5	1922.7	2089.1
1984	2436.1	1184.8	1635.8	368.9	624.0	1219.4	253.3	160.9	849.7	1944.9	671.1	144.3	2036.1	2207.9
1985	2642.2	1213.0	1793.6	386.6	666.1	1338.4	252.0	170.4	887.9	1998.1	716.1	147.6	2162.0	2333.3
1986	2858.2	1237.3	1971.1	404.6	715.3	1475.5	249.9	180.8	933.6	2050.9	763.2	150.8	2299.5	2465.7
1987	3081.3	1257.6	2170.2	423.0	771.2	1629.0	247.5	192.0	986.2	2103.5	811.9	153.8	2448.1	2605.5
1988	3313.2	1273.9	2393.2	441.7	833.8	1797.2	245.4	204.0	1044.7	2155.2	862.2	156.4	2608.2	2753.2
1989	3556.5	1286.1	2642.6	460.6	902.8	1979.9	243.7	216.7	1108.4	2205.5	914.4	158.8	2780.3	2909.1
1990	3813.3	1293.9	2921.6	479.7	978.4	2178.1	242.3	230.2	1176.9	2254.0	969.0	160.7	2965.3	3073.7
1991	4084.5	1296.7	3233.4	499.0	1060.7	2393.2	240.8	244.5	1249.6	2300.4	1026.0	162.2	3163.7	3247.6

WORLD

Year	Production	Consumption	Price	Scrap
1980	7064.8	6130.3	29.925	1065.7
1981	7524.9	6444.0	30.970	1110.0
1982	8022.2	6782.8	32.564	1154.2
1983	8581.9	7134.7	34.631	1199.2
1984	9213.4	7501.3	36.919	1246.6
1985	9911.2	7888.2	39.318	1296.9
1986	10667.7	8297.6	41.890	1350.6
1987	11481.8	8728.4	44.756	1407.7
1988	12357.9	9178.8	47.993	1468.4
1989	13303.2	9648.3	51.622	1533.2
1990	14323.9	10138.0	55.647	1602.4
1991	15426.1	10649.5	60.082	1676.5

ZINC
SIMULATION # 4

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	1756.9	1070.3	1156.4	299.5	516.1	859.6	276.1	130.3	762.0	1674.6	520.3	127.3	1644.4	1768.9
1981	1889.5	1094.9	1252.0	316.4	534.3	921.8	266.3	136.8	773.4	1721.3	548.0	131.7	1719.7	1870.0
1982	2026.2	1123.9	1360.3	333.8	555.2	996.3	258.6	144.1	789.8	1771.1	579.3	136.4	1804.0	1976.6
1983	2181.3	1155.5	1482.0	351.4	581.0	1077.6	255.2	152.0	811.7	1818.8	614.9	140.7	1899.4	2089.1
1984	2359.2	1187.2	1618.9	369.0	613.2	1170.3	254.0	160.5	840.4	1863.2	654.5	144.5	2007.2	2207.9
1985	2553.9	1216.4	1773.0	386.9	652.6	1279.4	252.7	169.8	876.6	1905.4	697.3	148.0	2126.9	2333.3
1986	2758.2	1241.9	1946.2	405.0	698.9	1406.1	250.6	180.1	920.2	1946.7	742.2	151.2	2257.7	2465.7
1987	2969.3	1263.4	2140.5	423.4	751.9	1548.6	248.1	191.2	970.6	1987.2	788.6	154.3	2399.2	2605.5
1988	3188.6	1281.1	2358.0	442.2	811.2	1705.2	245.9	203.0	1027.0	2026.4	836.6	157.2	2551.5	2753.2
1989	3419.0	1294.8	2601.3	461.2	876.9	1875.8	244.2	215.6	1088.5	2063.5	886.6	159.6	2715.3	2909.1
1990	3662.2	1304.2	2873.3	480.4	949.1	2061.1	242.7	229.0	1154.7	2098.1	938.7	161.7	2891.3	3073.7
1991	3919.2	1308.7	3177.3	499.9	1027.7	2262.8	241.2	243.1	1225.2	2130.1	993.3	163.4	3080.1	3247.6

WORLD

Year	Production	Consumption	Price	Scrap
1980	6632.2	5930.4	30.436	1044.8
1981	6983.2	6192.9	31.514	1085.9
1982	7373.4	6482.2	33.035	1126.5
1983	7825.5	6785.1	34.943	1167.4
1984	8347.4	7102.6	37.001	1209.8
1985	8933.0	7439.2	39.107	1254.6
1986	9574.0	7796.7	41.320	1301.9
1987	10268.1	8173.7	43.757	1351.8
1988	11018.9	8568.3	46.494	1404.7
1989	11831.8	8979.6	49.554	1460.8
1990	12712.0	9408.3	52.941	1520.6
1991	13664.1	9855.6	56.667	1584.3

ZINC
SIMULATION # 5

Year	North America Supply	North America Demand	South America Supply	South America Demand	Asia Supply	Asia Demand	Africa Supply	Africa Demand	Europe Supply	Europe Demand	Oceania Supply	Oceania Demand	Centrally Planned Supply	Centrally Planned Demand
1980	1854.0	1074.0	1173.1	299.6	520.6	976.8	271.9	129.6	768.1	1717.9	541.8	127.4	1668.5	1768.9
1981	2017.8	1097.2	1274.4	316.3	543.8	1073.1	264.2	136.7	782.8	1772.5	576.4	131.6	1756.3	1870.0
1982	2180.2	1123.6	1389.2	333.5	570.7	1183.6	257.2	144.7	803.7	1831.0	613.4	136.1	1853.4	1976.6
1983	2358.1	1152.2	1518.1	351.0	602.8	1301.2	253.8	153.0	830.8	1888.2	653.8	140.3	1961.9	2089.1
1984	2558.7	1180.9	1663.0	368.5	641.8	1430.2	252.6	161.8	864.8	1942.4	697.9	143.9	2083.5	2207.9
1985	2777.4	1207.2	1826.2	386.2	688.2	1576.7	251.4	171.5	906.3	1994.5	745.2	147.1	2218.1	2333.3
1986	3007.1	1229.7	2009.7	404.1	741.7	1742.6	249.5	182.1	955.3	2045.9	794.6	150.1	2364.8	2465.7
1987	3244.7	1248.0	2215.6	422.3	802.0	1926.6	247.2	193.4	1010.9	2097.0	845.7	152.8	2523.3	2605.5
1988	3491.5	1262.3	2446.3	440.8	868.8	2127.2	245.1	205.6	1072.3	2147.2	898.4	155.6	2693.8	2753.2
1989	3750.8	1272.4	2704.5	459.5	942.2	2344.3	243.5	218.4	1138.8	2195.7	953.3	157.4	2877.0	2909.1
1990	4024.6	1278.0	2993.4	478.5	1022.2	2578.9	242.1	232.0	1209.8	2242.2	1010.6	159.1	3073.8	3073.7
1991	4314.1	1278.5	3316.4	497.6	1109.0	2832.9	240.7	246.4	1285.2	2286.5	1070.7	160.3	3285.0	3247.6

WORLD

Year	Production	Consumption	Price	Scrap
1980	6798.0	6094.2	30.528	1064.4
1981	7215.6	6397.4	31.544	1108.6
1982	7667.6	6729.1	33.042	1152.5
1983	8179.3	7075.0	35.010	1196.9
1984	8762.4	7435.7	37.202	1243.3
1985	9412.8	7816.5	39.485	1292.5
1986	10122.6	8220.1	41.903	1344.8
1987	10889.2	8645.7	44.575	1400.4
1988	11716.3	9091.6	47.586	1459.4
1989	12610.1	9556.9	50.966	1522.2
1990	13576.5	10042.5	54.719	1589.3
1991	14621.2	10549.8	58.858	1661.0

ZINC
SIMULATION # 6

Year	North America		South America		Asia		Africa		Europe		Oceania		Centrally Planned	
	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand	Supply	Demand
1980	1879.2	1054.9	1177.8	298.3	535.5	1039.0	270.1	134.2	787.1	1709.0	553.3	125.9	1861.8	1768.9
1981	2048.5	1071.5	1280.8	314.8	564.3	1153.1	259.4	142.3	808.7	1762.3	590.0	129.9	1973.3	1870.0
1982	2219.6	1091.8	1397.2	331.8	596.8	1278.6	251.9	151.1	835.5	1818.8	629.1	134.1	2092.1	1976.6
1983	2409.0	1114.5	1527.8	349.0	634.7	1410.7	249.2	160.2	867.7	1873.2	671.5	137.9	2221.9	2089.1
1984	2621.6	1137.2	1674.7	366.1	679.4	1554.7	248.9	169.9	906.0	1924.4	717.6	141.1	2365.1	2207.9
1985	2851.8	1157.3	1840.1	383.4	731.2	1716.4	248.3	180.4	951.1	1973.6	766.6	143.9	2522.1	2333.3
1986	3092.4	1173.5	2026.0	401.0	790.0	1897.3	246.7	191.7	1002.8	2022.0	817.5	146.4	2692.3	2465.7
1987	3340.9	1185.5	2234.6	418.9	855.3	2096.0	244.6	203.8	1060.6	2069.9	870.0	148.8	2875.7	2605.5
1988	3599.1	1193.5	2468.3	437.0	926.9	2311.1	242.7	216.7	1123.8	2116.6	924.2	150.8	3072.8	2753.2
1989	3870.1	1197.1	2729.7	455.4	1005.1	2542.8	241.3	230.2	1192.0	2161.4	980.5	152.3	3284.5	2909.1
1990	4156.0	1196.0	3022.2	473.9	1089.8	2792.5	240.0	244.6	1264.7	2203.8	1039.4	153.4	3511.8	3073.7
1991	4458.1	1189.5	3349.4	492.5	1181.4	3062.3	238.6	259.8	1342.1	2243.7	1101.1	154.0	3755.5	3247.6

WORLD

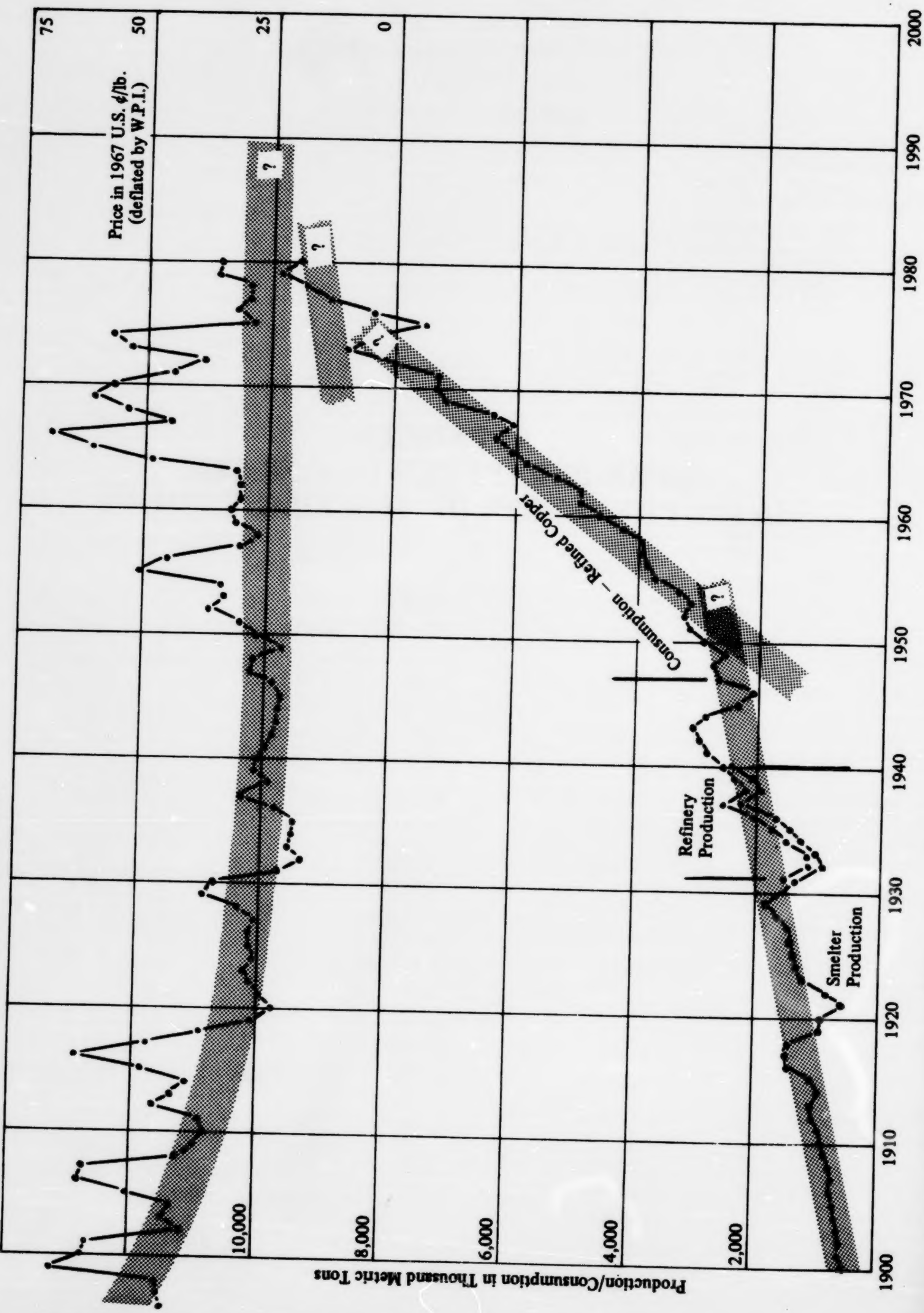
Year	Production	Consumption	Price	Scrap
1980	7064.8	6130.3	29.925	1065.7
1981	7524.9	6444.0	30.970	1110.0
1982	8022.2	6782.8	32.564	1154.2
1983	8581.9	7134.7	34.631	1199.2
1984	9213.4	7501.3	36.919	1246.6
1985	9911.2	7888.2	39.318	1296.9
1986	10667.7	8297.6	41.890	1350.6
1987	11481.8	8728.4	44.756	1407.7
1988	12357.9	9178.8	47.993	1468.4
1989	13303.2	9648.3	51.622	1533.2
1990	14323.9	10138.0	55.647	1602.4
1991	15426.1	10649.5	60.082	1676.5

TABLE 10-6
SIMULATION EQUATIONS
DEMAND
ZINC
(T-STATISTICS IN PARENTHESES)

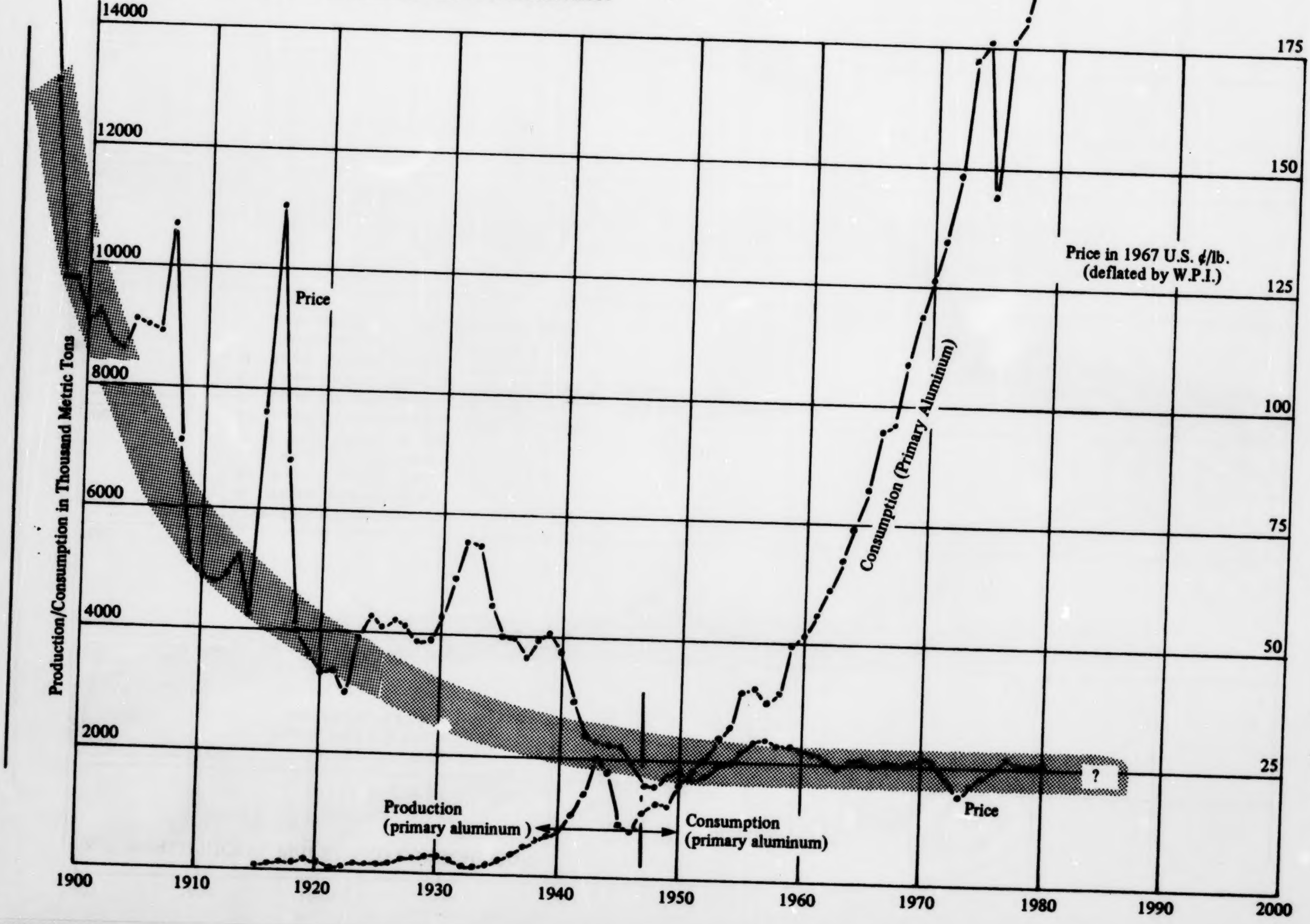
		North America	South America	Asia	Africa	Europe	Oceania	Centrally Planned
Ψ_0	Constant	44.5529	-384.534	-525.291		-239.372	-68.1776	
		(0.3241)	(-2.4679)	(-1.1813)		(-2.4221)	(-4.9736)	
Ψ_1	$P_{M,t}-P_{R,t}$	-2.8987						
		(-0.4529)						
Ψ_2	$P_{M,t-1}-P_{R,t-1}$	3.2183	-0.4698	-5.9813		-5.9412	-0.7599	
		(0.5877)	(-0.4336)	(-1.1433)		(-3.2647)	(-3.4063)	
Ψ_3	$S_{M,t-1}-D_{M,t-1}$	-0.0617	-0.0103	0.1954		-0.0641	-0.0113	
		(-0.6717)	(-0.5314)	(2.2111)		(-0.7181)	(-1.6917)	
Ψ_4	Y_t	4343.40	410.939	-138.384		2617.49	55.9709	
		(4.3450)	(2.1196)	(-0.0684)		(2.9331)	(0.7692)	
Ψ_5	Y_{t-1}	-3860.77	185.553	544.953		-1262.81	113.669	
		(-3.8497)	(0.5918)	(0.2299)		(-1.1551)	(1.3978)	
Ψ_6	$\dot{K}_{t-1}$	-618.125	-33.0869	609.878		560.746	27.5280	
		(-1.1369)	(-0.6330)	(1.4155)		(1.3412)	(0.7537)	
Ψ_7	$\dot{M}_{t-1}$	-2.0851	NA	NA		1.4573	0.0684	
		(-0.1151)				(0.3024)	(0.2803)	
Ψ_8	$Q^D_{M,t-1}$	0.5883	-0.4558	0.5036		-0.1176	-0.3237	
		(2.2688)	(-0.5905)	(1.3728)		(0.4011)	(-1.8943)	
	Dummy	-247.563	-39.9537	-304.438		-237.739	-36.3902	
		(-2.8101)	(-2.7306)	(-1.5378)		(-3.7576)	(-5.4583)	
	R^2	0.9309	0.9913	0.9783		0.9859	0.9651	
	SEE	71.4528	7.2848	47.6761		43.3874	5.6811	

APPENDIX: SOME PERSPECTIVES FOR THE INTERPRETATION OF THE SIMULATIONS

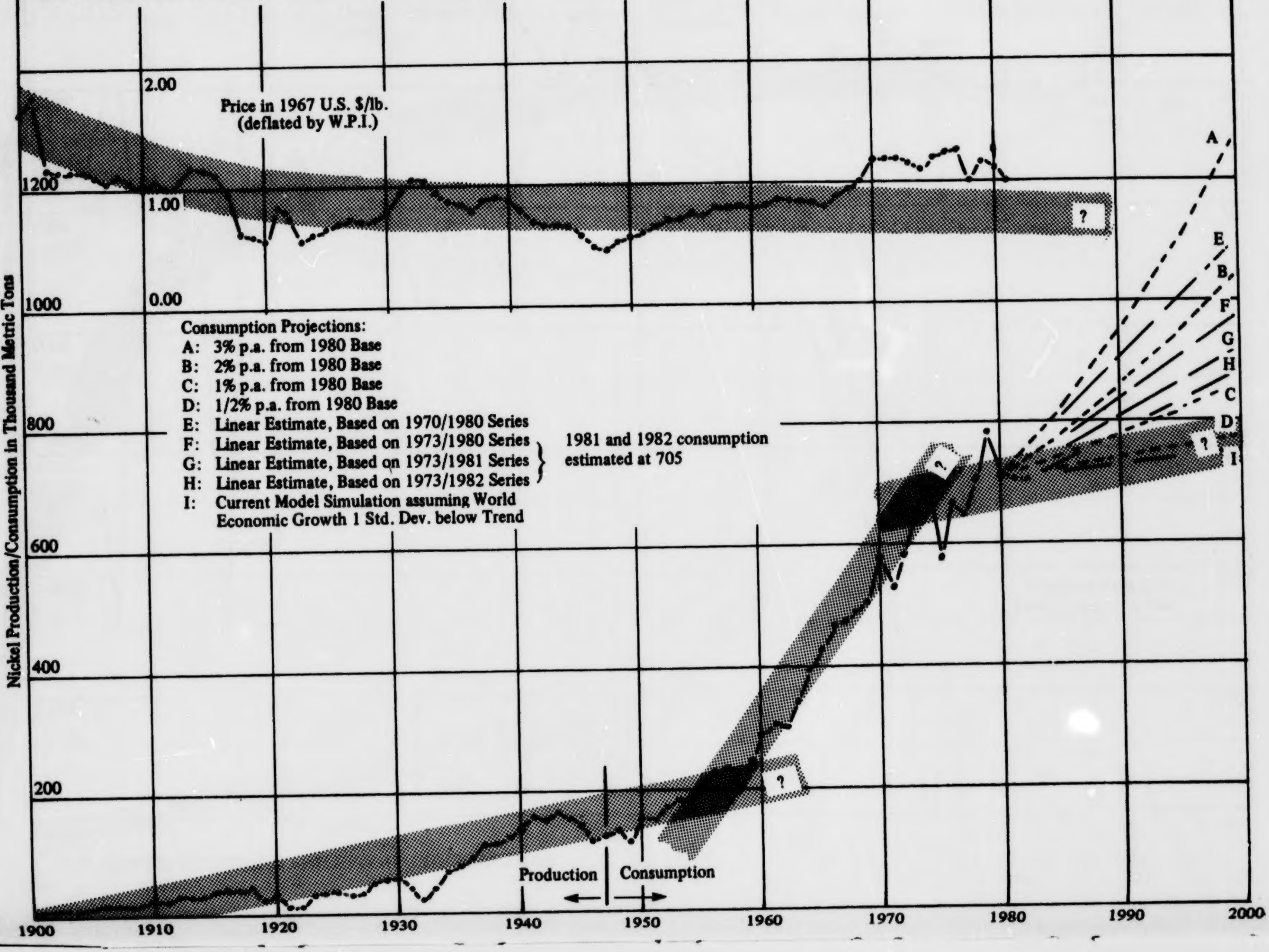
A-1: WORLD COPPER PRICES AND CONSUMPTION



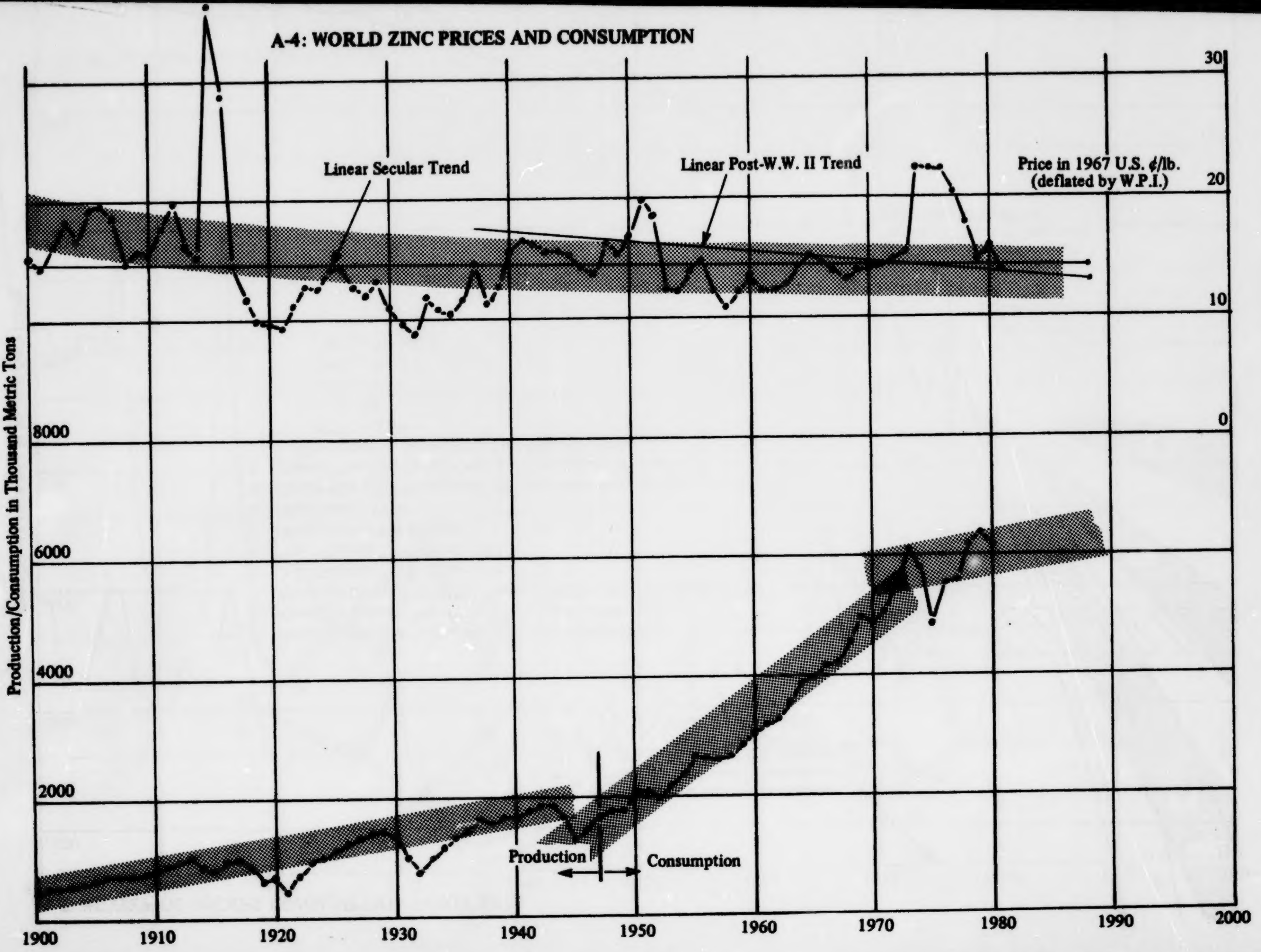
A-2: WORLD ALUMINUM PRICES AND CONSUMPTION



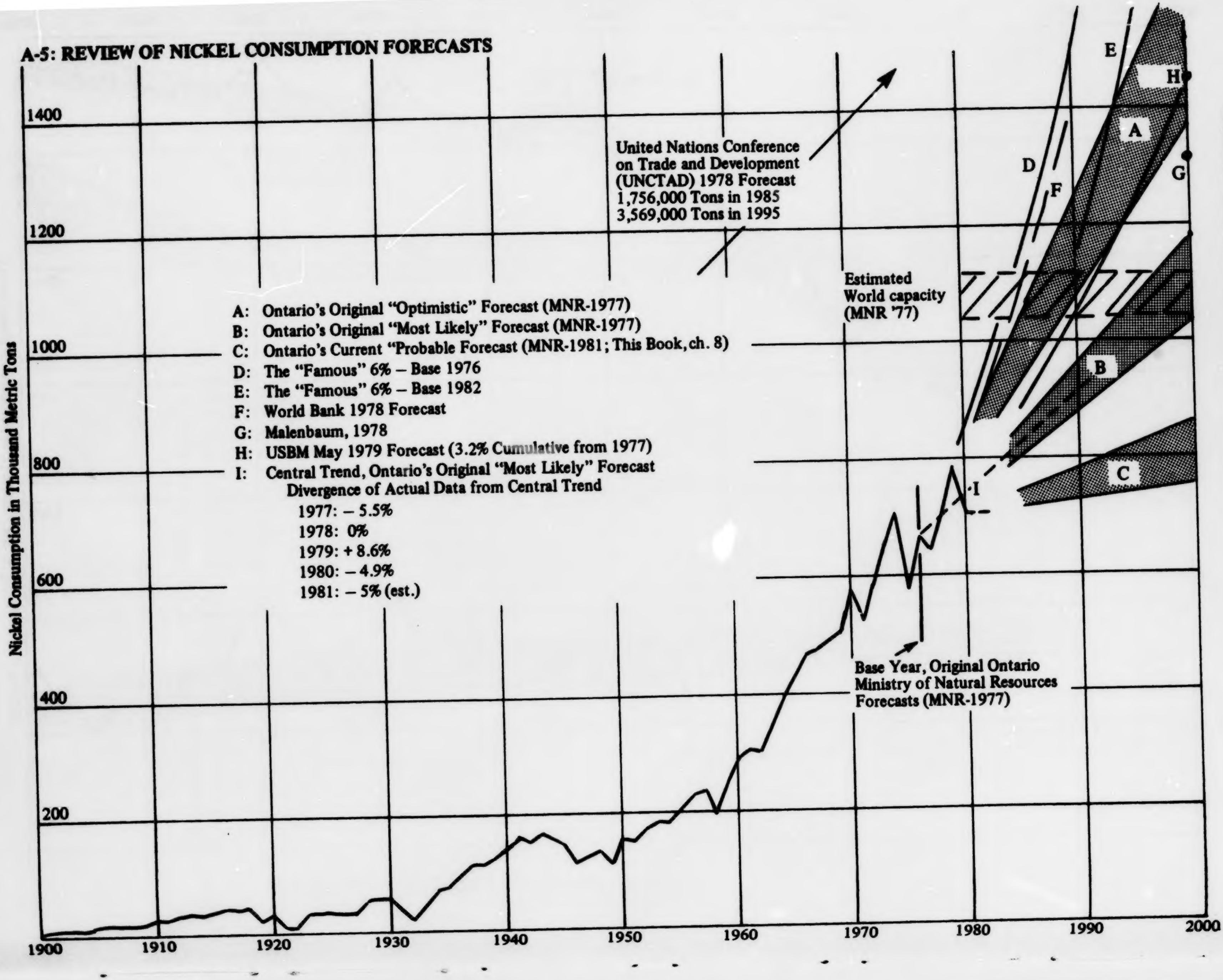
A-3: WORLD NICKEL PRICES AND CONSUMPTION



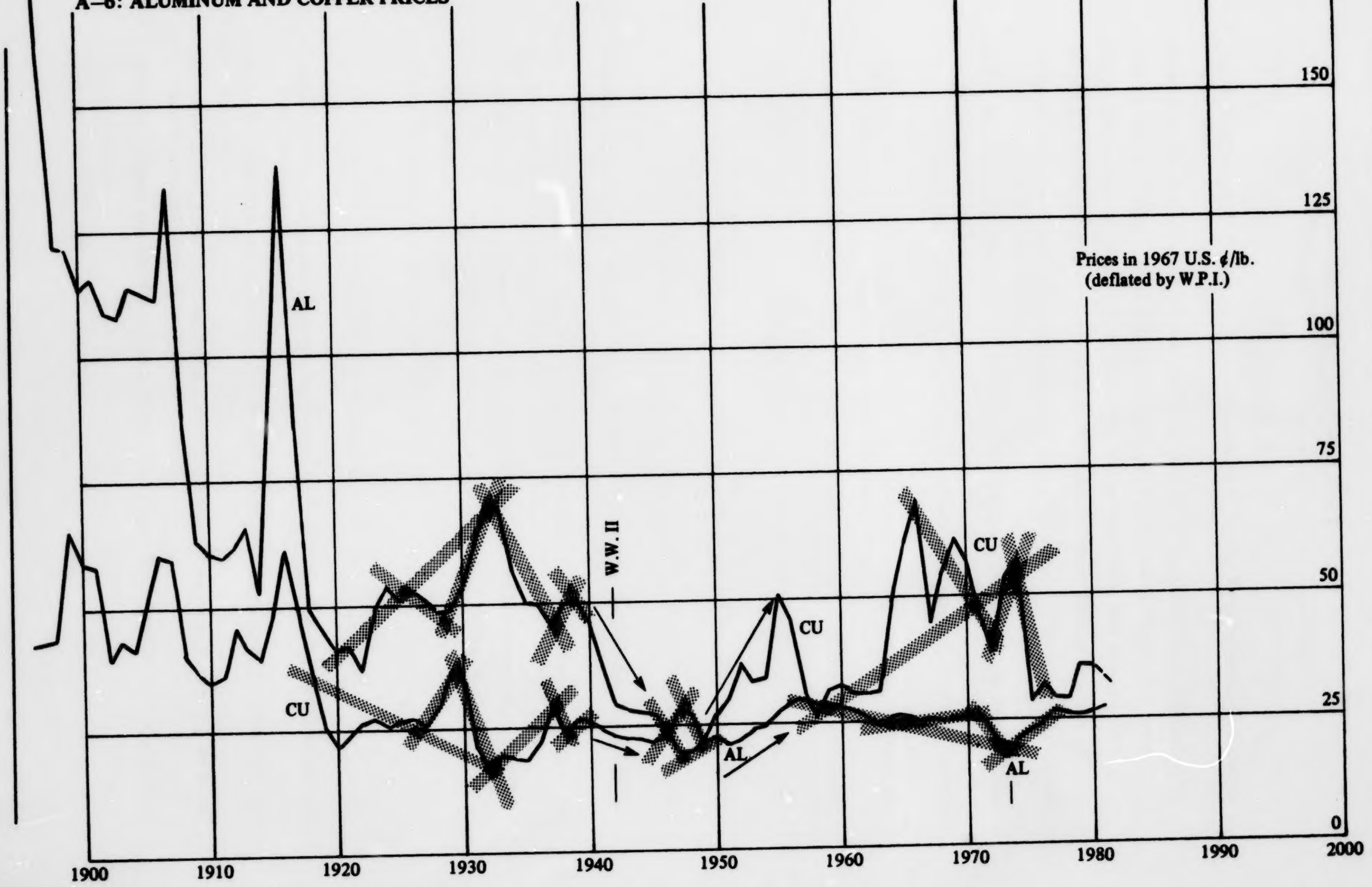
A-4: WORLD ZINC PRICES AND CONSUMPTION



A-5: REVIEW OF NICKEL CONSUMPTION FORECASTS



A–6: ALUMINUM AND COPPER PRICES



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